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ON ELLIPTIC PROBLEMS WITH ROUGH BOUNDARY DATA IN BESOV DISTRIBUTION SPACES

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We discuss applications of the Besov distribution spaces $B^{s}_{p,q}(\Omega)$ to a linear differential elliptic boundary-value problem

$$Au(x) \equiv \sum_{|\mu|\leq 2l} a_{\mu}(x)D^{\mu}u(x) = f(x) \quad \text{whenever} \quad x \in \Omega,$$

$$B_{j}u(x) \equiv \sum_{|\mu|\leq m_{j}} b_{j,\mu}(x)D^{\mu}u(x) = g_{j}(x) \quad \text{whenever} \quad x \in \Gamma,$$

$$j = 1, \dots l,$$

given in a bounded Euclidean domain Ω with infinitely smooth boundary Γ and having arbitrary distributions g_j in the right-hand sides of the boundary conditions. The elliptic PDO A is of even order $2l \geq 2$, whereas each boundary PDO B_j is of order $m_j \geq 0$. All coefficients a_{μ} and $b_{j,\mu}$ of these PDOs are infinitely smooth complex-valued functions on $\overline{\Omega}$ and Γ, respectively. We put $B := (B_1, \dots, B_l)$ and $m := \max\{m_1, \dots, m_l\}$. The case $m \geq 2l$ is possible.

As to the above spaces, we suppose that $0 < p < \infty$ and $0 < q < \infty$. Thus, we also involve quasi-normed spaces if $0 < p < 1$ and/or $0 < q < 1$. Given real numbers s and $\alpha > s - 2l$, we introduce the linear space

$$B^{s}_{p,q}(A, B^{\alpha}_{p,q}, \Omega) := \bigl\{u \in B^{s}_{p,q}(\Omega) : Au \in B^{\alpha}_{p,q}(\Omega)\bigr\}$$

endowed with the graph quasi-norm

$$\|u, B^{s}_{p,q}(\Omega)\| + \|Au, B^{\alpha}_{p,q}(\Omega)\|.$$

Here, Au is understood in the sense of the theory of distributions. This space is complete, and $C^{\infty}(\overline{\Omega})$ is dense in it.

Put

$$\pi(p, n) := \frac{1}{p} + \max\biggl\{0, (n-1)\biggl(\frac{1}{p} - 1\biggr)\biggr\},$$

where n is the dimension of Ω, with $n \geq 2$.

Theorem 1. *Let $0 < p < \infty$ and $0 < q < \infty$. Suppose that real numbers s and α satisfy the conditions*

$$s \leq m + \pi(p,n) \quad \text{and} \quad \alpha > m - 2l + \pi(p,n).$$

Then the mapping $u \mapsto (Au, Bu)$, with $u \in C^\infty(\overline{\Omega})$, extends uniquely (by continuity) to a bounded linear operator

$$(A,B) : B^s_{p,q}(A, B^\alpha_{p,q}, \Omega) \to B^\alpha_{p,q}(\Omega) \times \prod_{j=1}^{l} B^{s-m_j-1/p}_{p,q}(\Gamma).$$

This operator is Fredholm one. Its kernel $N \subset C^\infty(\overline{\Omega})$ and index do not depend on the parameters s, α, p, and q. Moreover, the range of this operator has a finite-dimensional complement $M \subset C^\infty(\overline{\Omega}) \times (C^\infty(\Gamma))^l$ that does not depend on these parameters as well.

The theorem 1 is applied to the elliptic problem with boundary data of arbitrarily low (specifically, negative) regularity (so called, rough data) provided that f is sufficiently regular. Moreover, f is allowed to have a certain negative regularity if $m \leq 2l - 1$ and $p \geq 1$. This theorem is proved in [1]. The case of normed Besov spaces, where $p > 1$ and $q > 1$, was covered in [2, Section 3] under assumption that $m \leq 2l - 1$ and $\alpha > 2l - 1$.

This theorem remains valid in the $q = \infty$ case excepting the density of $C^\infty(\overline{\Omega})$ in the space $B^s_{p,\infty}(A, B^\alpha_{p,\infty}, \Omega)$. In this case, the relevant Fredholm operator is considered as a restriction of any bounded operator from the theorem with $q < \infty$ and smaller s (cf. [3, Section 2]).

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3. *Мурач О. О., Чепурухіна І. С.* Еліптичні задачі з грубими крайовими даними у просторах Нікольського // Допов. Нац. акад. наук України. — 2021. — № 3. — С. 3–10.

ПРО ЕЛІПТИЧНІ ЗАДАЧІ З ГРУБИМИ КРАЙОВИМИ ДАНИМИ У ПРОСТОРАХ БЄСОВА РОЗПОДІЛІВ

Доповідь присвячена застосуванням просторів Бєсова $B^s_{p,q}$, де $s \in \mathbb{R}$ і $p, q \in (0, \infty)$, до еліптичних задач, у яких праві частини крайових умов є довільними розподілами. Такі задачі породжують нетерові обмежені оператори на відповідних парах просторів Бєсова як завгодно малого (зокрема, від'ємного) порядку s за умови, що права частина еліптичного рівняння є достатньо регулярним розподілом.