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# CONJUGATION PROBLEM WITH INITIAL-NONLOCAL CONDITIONS FOR MIXED FACTORIZED HIGHER ORDER EQUATIONS

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Let $\Omega^p = (\mathbb{R}/2\pi\mathbb{Z})^p$ be a $p$-dimensional torus, $\mathcal{D}^p = (-\alpha, \beta) \times \Omega^p$, $\mathcal{D}^p_- = (-\alpha, 0) \times \Omega^p$, $\mathcal{D}^p_+ = (0, \beta) \times \Omega_p$, where $p \in \mathbb{N}$, $\alpha$ and $\beta$ are positive real numbers.

The problem we aim to solve is finding a pair of functions $u_1 = u(t, x)$ and $u_2 = u_2(t, x)$, defined in $\mathcal{D}^p_-$ and $\mathcal{D}^p_+$, respectively, which satisfy the following differential equations

$$\begin{cases} \prod\limits_{j=1}^{n} \left(\frac{d^2}{dt^2} - \lambda_j^2 \Delta\right) u_1 = 0, \quad (t, x) \in \mathcal{D}^p_-, \\ \prod\limits_{j=1}^{m} \left(\frac{d}{dt} - \mu_j \Delta\right) u_2 = 0, \quad (t, x) \in \mathcal{D}^p_+, \end{cases} \tag{1}$$

with conjugate conditions

$$\lim_{t \to 0_-} \frac{d^{j-1} u_1}{dt^{j-1}} = \lim_{t \to 0_+} \frac{d^{j-1} u_2}{dt^{j-1}}, \quad j = 1, \ldots, m, \quad x \in \Omega^p, \tag{2}$$

nonlocal conditions

$$\left.\frac{d^{j-1} u_1}{dt^{j-1}}\right|_{t=-\alpha} - \nu_j \left.\frac{d^{j-1} u_2}{dt^{j-1}}\right|_{t=\beta} = \varphi_j(x), \quad j = 1, \ldots, m, \quad x \in \Omega^p, \tag{3}$$

and initial conditions

$$\left.\frac{d^{m+j-1} u_1}{dt^{m+j-1}}\right|_{t=-\alpha} = \varphi_{m+j}(x), \quad j = 1, \ldots, 2n - m, \quad x \in \Omega^p. \tag{4}$$

where $x = (x_1, \ldots, x_p)$, $\Delta = \frac{\partial^2}{\partial x_1^2} + \ldots + \frac{\partial^2}{\partial x_p^2}$, $n, m \in \mathbb{N}$, $1 \leq m \leq 2n$, $\lambda_j, \mu_j \in \mathbb{R}$, $(\lambda_j - \lambda_q)(\mu_j - \mu_q) \neq 0$ for $j \neq q$, $\nu_j \in \mathbb{C}$, $\varphi_j(x)$ are given functions.

In general, this problem are conditionally well-posed and its solvability is related with the problem of small denominators and may be unstable with respect to small variations in the coefficients of the problem and in the parameters of the domain. Using the Fourier method of separation of variable and metric approach [1,2], we will be discuss the conditions for the solvability of the problem (1)–(4) in Sobolev spaces and the proving estimates for small denominators for almost all (with respect to the Lebesgue measure in space $\mathbb{R}^m$) vectors $(\mu_1, \dots, \mu_m)$.

1. Ptashnyk B.Yo., Il'kiv V.S., Kmit' I.Ya. Polishchuk V.M. Nonlocal boundary value problems for partial differential equations. Naukova Dumka, Kyiv, 2002. (in Ukrainian)

2. Il'kiv V.S., Ptashnyk B.I. *Problems for partial differential equations with nonlocal conditions. Metric approach to the problem of small denominators.* Ukrainian Mathematical Journal 2006, **58**, 1847–1875.

## ЗАДАЧА СПРЯЖЕННЯ З ПОЧАТКОВО-НЕЛОКАЛЬНИМИ УМОВАМИ ДЛЯ МІШАНИХ ФАКТОРИЗОВАНИХ РІВНЯНЬ ВИСОКОГО ПОРЯДКУ

*Із використанням метричного підходу досліджуються умови єдиності та існування розв'язку у просторах Соболєва задачі спряження з початково-нелокальними умовами для мішаних факторизованих рівнянь високого порядку у циліндричній області.*