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REMOVABLE ISOLATED SINGULARITIES FOR SOLUTIONS OF ANISOTROPIC EVOLUTION P-LAPLACIAN EQUATION

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Let us consider solutions of the quasilinear parabolic equation in the divergent form

$$u_t - div\, A(x,t,u,\nabla u) = b(x,t,u,\nabla u), \quad (x,t) \in \Omega_T \setminus (x_0, 0), \tag{1}$$

satisfying a initial condition

$$u(x,0) = 0 \qquad x \in \Omega \setminus \{x_0\}, \tag{2}$$

where Ω is a bounded domain in R^n, $n \geq 3$, $x_0 \in \Omega$, $\Omega_T := \Omega \times (0,T)$, $0 < T < \infty$.

We suppose that the functions $A : \Omega_T \times R \times R^n \longrightarrow R^n$ and $b : \Omega_T \times R \times R^n \longrightarrow R^n$ are such that $A(\cdot,\cdot,u,\varsigma)$, $b(\cdot,\cdot,u,\varsigma)$ are Lebesque measurable for all $u \in R, \varsigma \in R^n$, and $A(x,t,\cdot,\cdot)$, $b(x,t,\cdot,\cdot)$ are continuous for almost all $(x,t) \in \Omega_T$, $A = (a_1, a_2, ...a_n)$. We also assume that the following structure conditions are satisfied:

$$\sum_{i=1}^{n} a_i(x,t,u,\varsigma)\varsigma_i \geqslant \nu_1 \sum_{i=1}^{n} |\varsigma_i|^{p_i},$$

$$|a_i(x,t,u,\varsigma)| \leqslant \nu_2 \left(\sum_{j=1}^{n} |\varsigma_j|^{p_j}\right)^{1-\frac{1}{p_i}}, \quad i = \overline{1,n}, \tag{3}$$

$$|b(x,t,u,\varsigma)| \leqslant \nu_2 \sum_{i=1}^{n} |\varsigma_i|^{p_i\left(1-\frac{1}{p}\right)}$$

with some positive constant ν_1, ν_2.

We further suppose that the following conditions are satisfied:

$$2n/(n+1) < p_1 \leqslant p_2 \leqslant ....p_n, \ \max_{1\leqslant i\leq n} < 2 + \frac{\kappa}{n} \tag{4}$$

where some p_i can be less than 2 (so called “singular” case), the other p_i can be greater than 2 (so called “degenerate” case), and

$$k = n(p-2) + p, \quad \frac{1}{p} = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{p_i}. \tag{5}$$

Let us denote by $D(r), r > 0$ the following set

$$D(r) = \left\{ (x,t) \in \Omega_T : \sum_{i=1}^{n} \left(\frac{|x_i - x_i^{(0)}|}{r^{k_i}} \right)^{p_i} + \frac{t}{r^k} \leq 1 \right\}, \tag{6}$$

where

$$k_i = \frac{p + n(p - p_i)}{p_i}. \tag{7}$$

We formulate the removability result in the term of behavior of the function

$$M(r) = \operatorname{ess\,sup}\{|\ u(x,t)\ | : (x,t) \in D(R_0) \setminus D(r)\}, \tag{8}$$

where R_0 is some sufficiently small fixed positive number such that $D(R_0) \subset \Omega_T$. It follows from [1] that $M(r)$ is finite number for any $r > 0$.

Let’s formulate the main results:

Theorem 1. *[2] Assume that conditions (3),(4) are fulfilled. Let u be a weak solution of the problem (1), (2). Then the singularity of solution u at the point $(x_0, 0)$ is removable if condition*

$$\lim_{r \to 0} M(r) r^n = 0. \tag{9}$$

is satisfied.

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