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NUMERICAL RESEARCH OF ATYPICAL GAS FILTRATION

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In the nature porous environments are carriers of hydrocarbons, water and many other useful substances. Adequate mathematical models of porous environments allow to construct algorithms for their optimal design and operation [1]. As a rule, mass transfer in porous media is described with nonlinear differential equations in partial derivatives or systems of such equations. Since the input parameters values and the properties of porous medium are usually known with low accuracy, the accurate solution of the modeled mathematical-physical problems shows considerable computing difficulties. Currently, a large number of fractional-differential models of diffusion-type transfer processes have been proposed [1, 2].
Mass transfer process in the porous medium is considered on the example of gas and fluid filtration described by the equation with the fractional derivative in time.

$$\frac{\partial}{\partial x}\left(\frac{kh}{\mu z}\frac{\partial p^l}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{kh}{\mu z}\frac{\partial p^l}{\partial y_1}\right)+\frac{\partial}{\partial z}\left(\frac{kh}{\mu z}\frac{\partial p^l}{\partial y_2}\right)=2mh\left(\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{p}{z}\right)+2qp_{at}\right).$$

A fractional derivative $\frac{\partial^\alpha p}{\partial t^\alpha}$ is decomposable according to Grünwald-Letnikov scheme

$$\frac{\partial^\alpha}{\partial t^\alpha}p(t)=\sum_{j=0}^{i}c_j^{(\alpha)}p(t_{i-j})-\sum_{k=0}^{m}\frac{(t_i)^{k-\alpha}}{\Gamma(k-\alpha+1)}p(t_k)\,,$$

$$F\left(\tilde{p},k,h,\mu,z\right)=-\frac{\partial}{\partial x}\left(\frac{kh}{\mu z}\cdot\tilde{p}\right)\cdot\frac{\partial\tilde{p}}{\partial x}-\frac{\partial}{\partial y}\left(\frac{kh}{\mu z}\cdot\tilde{p}\right)\cdot\frac{\partial\tilde{p}}{\partial y}\,.$$

Finding a generalized solution of problem consists in minimization of the functional

$$F(p)=\int_\Omega\sum_{k_1,k_2=1}^{2}a_{k_1k_2}\frac{\partial p}{\partial x}\frac{\partial p}{\partial y}dxdy+\int_\Omega dp^2dxdy-2\int_\Omega fpdxdy\,.$$

To avoid such a conflict at each iteration in the Grunwald scheme, the permeability coefficients in the geometric zones of the largest differences in gas pressure are corrected:

$$k_*(x,y) = \mu \Delta l \Delta Q / \Delta S \Delta p$$

in order to fulfill the condition

$$\lim_{k \to k_*} (M(t_0) - M(t_i) - \sum_{j=1}^{i} M_{ex}(t_j)) = 0 .$$

for each time point t_i. Here M – mass of calculated gas in the storage; M_{ex} – mass of extracted gas; $\Delta l, \Delta S$ – the length and cross-sectional area of the formation element through which the gas passes; $\Delta Q, \Delta p$ – the difference of gas consumption and gas pressure respectively.

Approbation of the method of adaptation of the gas permeability coefficient was performed on the following numerical experiment. The gas drowning coefficient is selected according to the described algorithm in the areas of the included wells $k \in (0.8 \cdot 10^{-13} m^2, 8.8 \cdot 10^{-12} m^2)$. The coefficient of the fractional derivative of the gas pressure over time was chosen in the range: $\alpha \in (0.94,..., 0.99)$.

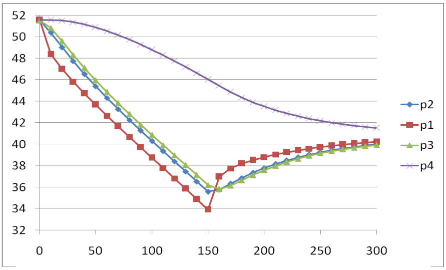

Fig 1. The dynamics of gas pressure for $\alpha = 0.99$*, k =0.15e-12*

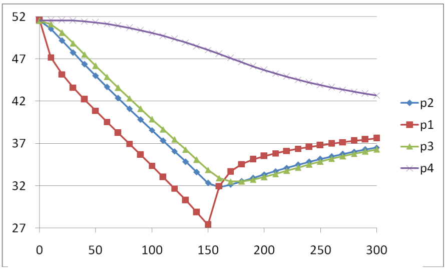

Fig 2. The dynamics of gas pressure for $\alpha = 0.986$*, k =0.15e-12*

The approach used in the work to fractional-differential filtration models is phenomenological, so the possibility of their application in each case should be justified using experimental data confirming the validity of the corresponding fractional-differential generalizations.

1. *Lopuh N. B, Pyanylo Ya. D.* Numerical analysis of models with fractional derivatives for gas filtration in porous media // Journal of Coupled Systems and Multiscale Dynamics – 2014. – Vol. 2 (1). – P. 15–19.
2. *Cook, Robert D.* Concept and Applications of Finite Element Analysis, fourth edition, John Wiley & Sons, 2002.

ЧИСЛОВЕ ДОСЛІДЖЕННЯ НЕТИПОВОЇ ФІЛЬТРАЦІЇ ГАЗУ

Запропоновано та досліджено нову чисельну модель нетипової фільтрації газу в пористому середовищі. Фільтрація газу моделюється за допомогою нелінійного диференціального рівняння в частинних похідних з дробовим порядком. Обговорюються методи розв'язування рівняння. Проведено аналіз та висновки на основі комп'ютерного експерименту.