The Conference of Young Scientists «Pidstryhach Readings – 2022», May 25–27, 2022, Lviv

UDC 539.3

PROBLEM WITH INTEGRAL CONDITIONS FOR SYSTEM OF PARTIAL DIFFERENTIAL EQUATIONS OF SECOND ORDER

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Let K_L be a class of quasi-polynomials. In the form $\phi(x) = \sum_{i=1}^{n} Q_i(x)\exp[\alpha_i x]$, where $Q_i(x)$ are given polynomials, $\alpha_i \in L \subseteq R$, $\alpha_l \neq \alpha_k$ for $l \neq k$. Each quasi-polynomial $\phi(x)$ defines a differential operator $\phi\left(\frac{\partial}{\partial\lambda}\right)\Phi(\lambda)\Big|_{\lambda=0} = \sum_{i=1}^{n} Q_i\left(\frac{\partial}{\partial\lambda}\right)\Phi(\lambda)\Big|_{\alpha_i}$ of finite order in the class of certain function $\Phi(\lambda)$.

In the strip $\Omega = \{(t,x) \in R^2 : t \in (T_1,T_2) \cup (T_3,T_4), x \in R\}$ we consider system of equations

$$\frac{\partial^2 U_i}{\partial t^2} + \sum_{j=1}^{n} a_{ij}\left(\frac{\partial}{\partial x}\right)\frac{U_i}{\partial t} + \sum_{j=1}^{n} b_{ij}\left(\frac{\partial}{\partial x}\right)U_j(t,x) = 0, \qquad i = 1,\ldots,n, \qquad (1)$$

satisfies conditions

$$\int_{T_1}^{T_2} t^k U_i(t,x)dt + \int_{T_3}^{T_4} t^k U_i(t,x)dt = \phi_i(x), \qquad k = 0,1, \qquad (2)$$

where $a_{ij}\left(\frac{\partial}{\partial x}\right)$, $b_{ij}\left(\frac{\partial}{\partial x}\right)$ are differentia expressions, with analytical symbols $a_{ij}(\lambda), b_{ij}(\lambda)$. Let be $\eta(\lambda) = \int_0^T W^{(n-1)}(t,\lambda)dt$ is a certain function, $W(t,\lambda)$ is a solution of the equation $L\left(\frac{d}{dt},\lambda\right)W(t,\lambda) \equiv 0$, satisfies conditions $W^{(n-1)}(t,\lambda)\big|_{t=0} = 1$, $W^{(n-2)}(t,\lambda)\big|_{t=0} = 0, \ldots, W(t,\lambda)\big|_{t=0} = 0$.

Denote by

$$P = \{\lambda \in C : \eta(\lambda) = 0\}\,. \tag{3}$$

Theorem. *Let* $\phi_i(x) \in K_M, i = 1,...,n$*, then the class* $K_{M \setminus P}$ *exist and unique solution of the problem (1), (2), where* P *is set (3), can be represented in the form*

$$U_i(t,x) = \sum_{i=1}^{n} \phi_i\left(\frac{\partial}{\partial\lambda}\right)\left\{\frac{1}{\eta(\lambda)}\overset{\sim T}{l}\left(\frac{d}{dt},\lambda\right)W(t,\lambda)\exp[\lambda x]\right\}\Bigg|_{\lambda=0},$$

where $\overset{\sim T}{l}\left(\frac{d}{dt},\lambda\right)$ *is transpose of a matrix.*

Be means of the differential-symbol method [1] we construction the solution of the problem (1), (2). This problem is a continues works [2,3].

1. *Kalenyuk P.I., Nytrebych Z.M,* Generalized Scheme of Separation of Variables. Differential-Symbol Method. Publishing House of Lviv Polytechnic Natyonaly University, 2002. – 292 p. (in Ukrainian).
2. *Kalenyuk P.I.,Kuduk G., Kohut I.V., Nytrebych Z.M,* Problem with integral conditions for differential operator equation // J. Math. Sci. – 2015. – 208, No. 3. – P.267–276.
3. *Kalenyuk P.I., Nytrebych Z.M., Kohut I.V., Kuduk G,* Nonlocal problem for partial differentia equations of higher order // Seventeenth International Scientific Mykhailo Kravchuk Conference. – 19 – 20 May, 2016, Kyiv, Conference Materials I. Differential and integral equations and its applications. 17. p

ЗАДАЧА З ОДНОРІДНИМИ ІНТЕГРАЛЬНИМИ УМОВАМИ ДЛЯ НЕОДНОРІДНОЇ СИСТЕМИ ДИФЕРЕНЦІАЛЬНИХ РІВНЯНЬ ІЗ ЧАСТИННИМИ ПОХІДНИМИ

За допомогою диференціально-символьного методу подано розвязок задачі з інтегральними умовами для системи диференціальних рівнянь із частинними похідними . Цей розвязок існує і єдиний в класі квазімногочленів.