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GOVERNING EQUATIONS OF THE PLANE ELASTICITY PROBLEM FOR A MULTILAYER INHOMOGENEOUS CYLINDER

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The efficiency of engineering design for segments and units of power plants and technical structures operating under thermal and force impacts calls for a better evaluation of their operational performance, particularly when entering critical loading ranges. The reliability in predicting the critical parameters of the operating performance depends significantly on the structural features affecting the level and character of the stress-strain profiles in structural members. One of the dominant constructional features is the material inhomogeneity that substantially affects the overall performance of a structure. Some kinds of spatial material inhomogeneity can be a result of structural intention, especially when using composite materials. With this regard, we can mention various multilayer composites consisting of a number of layers of dissimilar materials assembled in order to face specific operational requirements. In many practical cases, the individual layers can be regarded as homogeneous. Thus, under sufficient geometric and physical conditions (e.g., if the layers are thin enough compared to the thickness of the entire structure, etc.), a corresponding homogenization procedure can be employed to estimate the constant macro-scale material properties of the assembly [1]. This allows for significant simplification of a general analysis of their operational performance.

In some cases, however, either the layers' geometry or their specific material features are insufficient for implementing a homogenization routine to encounter the layering adequately. This is an issue, e.g., for multilayer functionally graded materials composing both homogeneous and inhomogeneous layers whose thickness is comparable with the thickness of the composite structure [2]. The analysis of such composites implies the use of a macroscopic inhomogeneity model which complicates both the analytical and numerical solution attempts due to the fact that the constitutive equations necessarily involve variable coefficients which often are a priori unknown [3]. For the analysis of latter structures, a single layer method is often employed implying the solution of corresponding elasticity problem for every individual layer with account for its specific geometry and material properties. A solution for each layer is to possess a sufficient number for freedom degrees in order to comply with the interface conditions and the boundary conditions on the limiting surfaces of the composite. Despite the apparent efficiency of such approach, its application is quite limited for composites consisting of a greater number of layers or if the material properties of all or some layer are functions of the spatial coordinates.

In the latter case, the single solid approach seems to be a more advanced tool. Within the framework of this approach, the entire composite assembly is considered as an inhomogeneous structure with stepwise variation of the material properties across the layering. In view of the latter feature of the material profiles, the classical models of macro-mechanics can no longer be employed as they imply spatial differentiation. A new model is to be based on the generalized differentiation. Moreover, an efficient solution method is to be used within the framework of such model in order to analyze a spatially inhomogeneous structure.

In [4], a technique based on the single solid approach was developed for solving a one-dimensional elasticity problem for a multilayer inhomogeneous hollow cylinder. The technique involves the application of the direct integration method along with the apparatus of generalized differentiation.

This talk addresses the extension of the latter technique towards the two-dimensional elasticity problem for a multilayer inhomogeneous hollow cylinder subject to non-axisymmetric force loading on its inner and outer circumferences.

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КЛЮЧОВІ РІВНЯННЯ ПЛОСКОЇ ЗАДАЧІ ТЕОРІЇ ПРУЖНОСТІ ДЛЯ БАГАТОШАРОВОГО НЕОДНОРІДНОГО ЦИЛІНДРА

Виведено ключові рівняння плоскої задачі теорії пружності для багатошарового порожнистого циліндра з пружними властивостями шарів, залежними від радіальної координати. З використанням подання макроскопічних властивостей циліндра у вигляді кусково-змінної функції та із застосуванням апарату узагальненого диференціювання ключові рівняння зведено до інтегральних рівнянь для компонент тензора напружень, розв'язок яких знаходиться з використанням методу резольвент.