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SELF ORTHOGONAL TERNARY MEDIAL QUASIGROUPS

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A ternary operation defined on a set Q is a quasigroup, if it is i-invertible for all $i \in 1, 2, 3$.

The definition of n-ary orthogonality from [1] for ternary operations is the following: a triplet of ternary operations f_1, f_2, f_3 defined on a set Q of order m is called *orthogonal*, if for all $a_1, a_2, a_3 \in Q$ the system of equations

$$\begin{cases} f_1(x_1, x_2, x_3) = a_1, \\ f_2(x_1, x_2, x_3) = a_2, \\ f_3(x_1, x_2, x_3) = a_3 \end{cases}$$

has a unique solution.

For every permutation $\sigma \in S_4$ a *σ-parastrophe* ${}^{\sigma}f$ of an invertible ternary operation f is defined by

$${}^{\sigma}f(x_{1\sigma}, x_{2\sigma}, x_{3\sigma}) = x_{4\sigma} :\Longleftrightarrow f(x_1, x_2, x_3) = x_4.$$

A σ-parastrophe is called:

- an *i-th division*, if $\sigma = (i4)$ for $i = 1, 2, 3$;
- *principal*, if $4\sigma = 4$.

Note that there are at most $3! = 6$ distinct principal parastrophes. The existence of all f's divisions requires i-invertibility of f for all $i \in \{1, 2, 3\}$. However, for existence of principal parastrophes an operation does not necessarily have the property of invertibility.

An operation is called *self orthogonal*, if all its distinct principal parastrophes are orthogonal. Note that if a ternary operation is self orthogonal, then we have a set of 6 triple-wise orthogonal operations. This notion for n-ary operations was studied by P. Syrbu (see for example [2], [3]). Here we will restrict consideration of this concept for ternary case.

A ternary quasigroup $(Q; f)$ is *medial* [4] if and only if there exists an abelian group $(Q; +)$ such that

$$f(x_1, x_2, x_3) = \gamma_1 x_1 + \gamma_2 x_2 + \gamma_3 x_3 + a, \tag{1}$$

where γ_1, γ_2, γ_3 are pairwise commuting automorphisms of $(Q; +)$ and $a \in Q$.

Theorem 1. *A ternary medial quasigroup* (Q, f) *with* (1)*, where* γ_1*,* γ_2*,* γ_3 *are pairwise different, is self orthogonal if and only if the mappings*

$$\begin{gathered}\gamma_1-\gamma_2, \qquad \gamma_1-\gamma_3, \qquad \gamma_2-\gamma_3, \qquad \gamma_1+\gamma_2+\gamma_3, \\ \gamma_1\gamma_2+\gamma_1\gamma_3+\gamma_2\gamma_3-\gamma_1^2-\gamma_2^2-\gamma_3^2\end{gathered} \tag{2}$$

are automorphisms of (Q, f)*.*

Recall the notion of strong orthogonality with the restrictions for ternary case and mediality of the given quasigroup, which follows from [5]: a triplet of ternary medial quasigroups are strongly orthogonal, if all minors of order s, where $s \in \{1, 2, 3\}$, of the corresponding determinant are invertible.

Theorem 2. *A ternary medial quasigroup* (Q, f) *is strongly self orthogonal if and only if the mappings* (2) *and the mappings*

$$\begin{array}{lll}\gamma_2\gamma_3-\gamma_1^2, & \gamma_1\gamma_3-\gamma_2^2, & \gamma_1\gamma_2-\gamma_3^2, \\ \gamma_1+\gamma_2, & \gamma_1+\gamma_3, & \gamma_2+\gamma_3\end{array}$$

are automorphisms of (Q, f)*.*

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САМООРТОГОНАЛЬНІ ТЕРНАРНІ МЕДІАЛЬНІ КВАЗІГРУПИ

Досліджується ортогональність тернарних медіальних квазігруп, тобто лінійних квазігруп, які мають властивість комутатування автоморфізмів їх розкладу над деякою абелевою групою. Розглядається випадок, коли така квазігрупа ортогональна до своїх головних парастрофів, тобто самоортогональність. Описано умови, коли тернарна медіальна квазігрупа є самоортогональною, та досліджено умови коли ця квазігрупа є сильно самоортогональною.