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Harnack's inequality for quasilinear elliptic equations with nonstandard growth conditions

Mariia Savchenko

Institute of Applied Mathematics and Mechanics NAS of Ukraine,
shan_maria@ukr.net

We consider quasilinear elliptic equations of the form

$$\operatorname{div}\left(g(x, |\nabla u|)\frac{\nabla u}{|\nabla u|}\right) = 0, \quad x \in \Omega, \tag{1}$$

where Ω is a bounded domain in $\mathbb{R}^n$, $n \geqslant 2$.

Throughout the paper we suppose that the function $g(x, \mathrm{v}) : \Omega \times \mathbb{R}_+ \to \mathbb{R}_+$, $\mathbb{R}_+ := [0, +\infty)$, satisfies the following assumptions:

(g) $g(\cdot, \mathrm{v}) \in L^1(\Omega)$ for all $\mathrm{v} \in \mathbb{R}_+$, $g(x, \cdot)$ is continuous and non-decreasing for almost all $x \in \Omega$, $\lim\limits_{\mathrm{v}\to+0} g(x, \mathrm{v}) = 0$ and $\lim\limits_{\mathrm{v}\to+\infty} g(x, \mathrm{v}) = +\infty$;

(g_1) there exist $c_1 > 0$, $q > 1$ and $b_0 \geqslant 0$ such that

$$\frac{g(x, \mathrm{w})}{g(x, \mathrm{v})} \leqslant c_1 \left(\frac{\mathrm{w}}{\mathrm{v}}\right)^{q-1}, \tag{2}$$

for all $x \in \Omega$ and for all $\mathrm{w} \geqslant \mathrm{v} \geqslant b_0$;

(g_2) there exists $p > 1$ such that

$$\frac{g(x, \mathrm{w})}{g(x, \mathrm{v})} \geqslant \left(\frac{\mathrm{w}}{\mathrm{v}}\right)^{p-1}, \tag{3}$$

for all $x \in \Omega$ and for all $\mathrm{w} \geqslant \mathrm{v} > 0$;

(g_3) for any $K > 0$ and for any ball $B_{8r}(x_0) \subset \Omega$ there exists $c_2(K) > 0$ such that

$$g(x_1, \mathrm{v}/r) \leqslant c_2(K)\, e^{\lambda(r)} g(x_2, \mathrm{v}/r),$$

for all x_1, $x_2 \in B_r(x_0)$ and for all $r \leqslant \mathrm{v} \leqslant K$. Here $\lambda(r) : (0, r_*) \to \mathbb{R}_+$ is a continuous, non-increasing function, satisfying the conditions described below.

Our main result is Harnack's inequality for bounded weak solutions to equation (1).

Теорема 1 (Harnack inequality)**.** *Fix a point* $x_0 \in \Omega$ *and consider the ball* $B_{8\rho}(x_0) \subset \Omega$*. Let conditions* (g), (g_1)–(g_3) *be fulfilled, and let* u *be a nonnegative bounded weak solution to Eq.* (1)*. Then there exist positive constants* C*,* c*,* β *depending only on the data such that*

$$\operatorname*{ess\,sup}_{B_\rho(x_0)} u \leqslant C\Lambda(c,\beta,\rho)\Big(\operatorname*{ess\,inf}_{B_\rho(x_0)} u + (1+b_0)\rho\Big), \tag{4}$$

where $\Lambda(c,\beta,\rho) := \exp\Big(c\exp\big(\beta\lambda(\rho)\big)\Big)$.

1. *Maria O. Shan (Savchenko), Igor I. Skrypnik, Mykhailo V. Voitovych;* Harnack's inequality for quasilinear elliptic equations with generalized Orlicz growth, Electronic Journal of Differential Equations, Vol. 2021 (2021), No. 27, pp. 1-16.