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Point spectrum in conflict dynamical systems with attractive interaction

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Let Ω be a compact set from $\mathbb{R}^d, d \geq 1$. The Borel σ-algebra and Lebesgue measure are denoted by $\mathcal{B}$ and λ, respectively. We suppose that Ω is subjected to iterative procedure of regionalization, which, in fact, coincides with typical construction of fractal divisions $\Omega = \bigcup_{i_1=1}^{n} \Omega_{i_1}, 2 \leq n < \infty, \Omega_{i_1} = \bigcup_{i_2=1}^{n} \Omega_{i_1 i_2}$, $\Omega = \bigcup_{i_1,i_2=1}^{n} \Omega_{i_1 i_2}, \ldots \Omega_{i_1 \ldots i_{k-1}} = \bigcup_{i_k=1}^{n} \Omega_{i_1 \ldots i_k}, \quad \Omega = \bigcup_{i_1,\ldots,i_k=1}^{n} \Omega_{i_1 \ldots i_k}, \ldots$ with condition: $|\Omega_{i_1 \ldots i_k}| = \lambda(\Omega_{i_1 \ldots i_k}) \longrightarrow 0, \ k \to \infty$ ([5]).

Compact Ω is considered as a conflict space (space of living resource) for a couple of alternative sides A and B which we call by opponents. We suppose that distributions of opponents along Ω at an initial time moment, $t = 0$, may be represented by probability measures μ, ν from some space $\mathcal{M}(\Omega)$ and intersection of their supports is nonempty,

$$\sup(\mu) \bigcap \sup(\nu) \neq \varnothing.$$

Some noncommutative binary map $\divideontimes$ in the space of probability measures $\mathcal{M}(\Omega)$ describe conflict interactions between A and B:

$$\mu^{t+1} = \mu^t \divideontimes \nu^t, \quad \nu^{t+1} = \nu^t \divideontimes \mu^t.$$

The sequential iteration of $\divideontimes$ with starting $\mu^0 = \mu, \ \nu^0 = \nu$ generates some trajectory at discrete time:

$$\{\mu^t, \nu^t\} \xrightarrow{\divideontimes, t} \{\mu^{t+1}, \nu^{t+1}\}, \quad t = 1, 2, \ldots \tag{1}$$

All measure $\mu^t, \nu^t, \ t \geq 1$ are uniformly distributed on regions $\Omega_{i_1 \ldots i_k}$, and are defined by beforehand given measures $\mu, \nu \in \mathcal{M}(\Omega)$ according to the iterative rule:

$$\mu^t(\Omega_{i_1 \ldots i_k}) = p_{i_1 \ldots i_k} := p_{i_1}^1 \cdot \ldots \cdot p_{i_k}^t,$$

$$\nu^t(\Omega_{i_1 \ldots i_k}) = r_{i_1 \ldots i_k} = r_{i_1}^1 \cdot \ldots \cdot r_{i_k}^t, \quad t = k,$$

with

$$p_i^t = \frac{p_i^{t-1}(1 + r_i^{t-1})}{1 + \theta^{t-1}}, \quad r_i^t = \frac{r_i^{t-1}(1 + p_i^{t-1})}{1 + \theta^{t-1}}, \quad t = 1, \ldots, \tag{2}$$

where $\theta^t = (\mathbf{p}^t, \mathbf{r}^t) = \sum_{i=1}^n p_i^t r_i^t$, $\sum_{i=1}^n p_i^t = \sum_{i=1}^n r_i^t = 1$ and $p_i^0 \equiv p_i = \mu(\Omega_i)$, $r_i^0 \equiv r_i = \nu(\Omega_i)$.

In [1] (see also [2, 4]) it was proven the existence of the limit invariant measures $\mu^\infty = \lim_{t\to\infty} \mu^t, \nu^\infty = \lim_{t\to\infty} \nu^t$. Here we find the sufficient conditions for $\mu^\infty, \nu^\infty \in \mathcal{M}_{\rm pp}$.

Let us introduce the values $\sigma_i^t := p_i^t + r_i^t, \quad \rho_i^t := p_i^t \cdot r_i^t, \quad t \geq 0.$

Theorem 1. *The limit measures μ^∞ and ν^∞, constructed in according with formulae (1)-(2) which describe the attractive interaction, are pure point, $\mu^\infty, \nu^\infty \in \mathcal{M}_{\rm pp}$, if there exists a single region $\Omega_{\mathbf{i}}$, such that one of condition*

$$p_{\mathbf{i}} = \max_{i=1}^{n}\{p_i\}, \quad r_{\mathbf{i}} = \max_{i=1}^{n}\{r_i\}, \tag{3}$$

or

$$\sigma_{\mathbf{i}} = \sigma_{\max} = \max_{i=1}^{n}\{\sigma_i\}, \quad \rho_{\mathbf{i}} = \rho_{\max} = \max_{i=1}^{n}\{\rho_i\}, \tag{4}$$

is fulfilled.

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Виникнення точкового спектру в динамічних системах конфлікту з притягальною взаємодією

Розглядається спектральна задача для розподілів динамічних систем конфлікту з притягальною взаємодією на просторах з ітераційним фрактальним подрібненням. Знайдено критерій у термінах початкових розподілів, який гарантує виникнення точкового спектру у граничних розподілах.