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Cauchy problem for nonlocal nonlinear Schrödinger equation with step-like initial data: asymptotics and inverse scattering

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We consider the initial value problem for integrable nonlocal nonlinear Schrödinger (NNLS) equation

$$iq_t(x,t) + q_{xx}(x,t) + 2q^2(x,t)\bar{q}(-x,t) = 0,\ x \in (-\infty,\infty),\ t \geq 0, \qquad (1)$$

$$q(x,0) = q_0(x), \qquad (2)$$

where the solution $q(x,t)$ satisfies the following step-like boundary conditions for all $t \geq 0$:

$$q(x,t) \to 0,\ x \to -\infty, \quad \text{and} \quad q(x,t) \to A,\ x \to \infty, \qquad (3)$$

with an arbitrary constant $A > 0$.

The NNLS equation was introduced by M. Ablowitz and Z. Musslimani in [1] as a nonlocal reduction of the classical integrable Ablowitz-Kaup-Newell-Segur (AKNS) system (a.k.a coupled Schrödinger equations):

$$iq_t + q_{xx} + 2q^2 r = 0, \qquad (4а)$$

$$-ir_t + r_{xx} + 2r^2 q = 0, \qquad (4б)$$

with $r(x,t) = \bar{q}(-x,t)$.

For studying the Cauchy problem (1)–(3) we apply the inverse scattering transform (IST) method in the form of the 2×2 matrix Riemann-Hilbert factorization problem, which allows us to “linearize” the initial nonlinear problem (i.e. we reduce it to a series of linear problems).

Moreover, we analyze the long-time behavior of the solution $q(x,t)$ in the physically interesting region $\frac{x}{t} = O(1)$, i.e. along the rays $\frac{x}{t} = const$. We show that the asymptotic picture involves two qualitatively different regions: decaying region for $x < 0$ and “modulated constant” region for $x > 0$ [2]. This is in sharp contrast with the classical local integrable equations, particularly with the local counterpart of the NNLS equation, the classical nonlinear Schrödinger equation (with $r(x,t) = \bar{q}(x,t)$ in (4)), where there always exists three different asymptotic zones: decaying region, modulated wave region and a transition elliptic wave region with straight line boundaries. Also we partially address the problem of description of the solution in the curved transition zones between decaying and constant regions, where we obtain the asymptotics of the solution along the curves $t = const \cdot x^{2-\alpha}$ with $\alpha \in (0, 1)$.

1. *M. Ablowitz, Z. Musslimani* Integrable nonlocal nonlinear Schrödinger equation // Phys. Rev. Lett. – 2013. – №110 – 064105.

2. *Ya. Rybalko, D. Shepelsky* Long-time asymptotics for the nonlocal nonlinear Schrödinger equation with step-like initial data // J. Differential Equations – 2021. – №270 – 694-724.

3. *Ya. Rybalko, D. Shepelsky* Curved wedges in the long-time asymptotics for the integrable nonlocal nonlinear Schrödinger equation // arXiv:2004.05987.