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GLOBAL SOLVABILITY, STABILITY AND DISSIPATIVITY OF TIME-VARYING SEMILINEAR DIFFERENTIAL-ALGEBRAIC EQUATIONS, AND APPLICATIONS

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Consider implicit differential equations

$$\frac{d}{dt}[A(t)x(t)] + B(t)x(t) = f(t, x(t)), \qquad t \geq t_+, \tag{1}$$

$$A(t)\frac{d}{dt}x(t) + B(t)x(t) = f(t, x(t)), \qquad t \geq t_+, \tag{2}$$

where $t_+ \geq 0$, $A, B\colon [t_+, \infty) \to L(\mathbb{R}^n)$ and $f\colon [t_+, \infty) \times \mathbb{R}^n \to \mathbb{R}^n$, and the initial condition $x(t_0) = x_0$ ($t_0 \geq t_+$). We do not require the operator $A(t)$ to be nondegenerate. Thus, in the general case, the operator $A(t)$ is degenerate and therefore these equations are called time-varying *differential-algebraic equations* (*DAEs*) or *degenerate differential equations* (they are also called *descriptor systems*). In the terminology of DAEs, equations of the form (1), (2) are commonly referred to as *semilinear*. Note that the operator $B(t)$ can also be degenerate. The operator pencil $\lambda A(t) + B(t)$ (λ is a complex parameter) which corresponds to the left side (the linear part) of the DAEs (1) and (2) is called characteristic. It is assumed that the pencil $\lambda A(t) + B(t)$ is a regular pencil of index not higher than 1, and A, $B \in C^1([t_+, \infty), L(\mathbb{R}^n))$. A function $x \in C([t_0, t_1), \mathbb{R}^n)$ is said to be a *solution of the equation* (1) *on* $[t_0, t_1)$ ($[t_0, t_1) \subseteq [t_+, \infty)$) if the function $A(t)x(t)$ is continuously differentiable on $[t_0, t_1)$ and $x(t)$ satisfies (1) on $[t_0, t_1)$. If we consider the equation (2), then its solution $x(t)$ has to be continuously differentiable on $[t_0, t_1)$.

In the present work, we obtain the theorems on the existence and uniqueness of global solutions, the Lagrange stability (the boundedness of global solutions), the dissipativity (the ultimate boundedness of solutions) and the Lagrange instability (solutions have finite escape time) for the time-varying semilinear DAEs (1), (2) [1,2]. Also, we obtain the theorems on the Lyapunov stability, asymptotic stability, complete stability (the asymptotic stability in the large) and the Lyapunov instability [2,3]. The Lagrange stability (the dissipativity) of a DAE means the existence of global solutions for all feasible initial values, i.e., for all consistent initial values, and the boundedness (the ultimate boundedness) of all solutions. Thus, in contrast to the Lyapunov stability, the Lagrange stability and the dissipativity of a DAE can be viewed, in a certain sense, as the stability of the entire DAE (i.e., the stability of all its solutions), not just of a separate solution analyzed for stability.

DAEs of the form (1), (2) are used to describe mathematical models in control theory, radio electronics, robotics, economics, ecology and chemical kinetics (see, e.g., [4–6]). The application of the theorems obtained in the present work to the study of certain mathematical models of electrical circuits with nonlinear and time-varying elements are discussed (the mathematical models are considered in [2, 3]).

In the present work, we employ the Barbashin-Krasovskii theorem on the complete stability of explicit ordinary differential equations, Lyapunov's second method and methods of J. La Salle and T. Yoshizawa based on it, and time-varying spectral projectors [7].

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ГЛОБАЛЬНА РОЗВ'ЯЗНІСТЬ, СТІЙКІСТЬ І ДИСИПАТИВНІСТЬ НЕСТАЦІОНАРНИХ НАПІВЛІНІЙНИХ ДИФЕРЕНЦІАЛЬНО-АЛГЕБРАЇЧНИХ РІВНЯНЬ ТА ЗАСТОСУВАННЯ

Отримано теореми про глобальну розв'язність, стійкість за Лагранжем (обмеженість усіх розв'язків), дисипативність (граничну обмеженість розв'язків) і нестійкість за Лагранжем (відсутність глобальних розв'язків) для нестаціонарних напівлінійних диференціально-алгебраїчних рівнянь (ДАР). Крім того, доведено теореми про стійкість, асимптотичну стійкість і нестійкість за Ляпуновим положення рівноваги нестаціонарних напівлінійних ДАР та про їх асимптотичну (повну) стійкість. В якості застосування розглянуто математичні моделі певних типів електричних кіл.