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NONLOCAL PROBLEM WITH INTEGRAL CONDITION FOR NONHOMOGENEOUS SYSTEM OF EVOLUTION EQUATIONS OF SECOND ORDER

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Let H be Banach space, let A be linear operator acting in it $A:H\to H$, and for this operator arbitrary powers $A^n, n=2,3,...$ be also defined in H. Denote be $x(\lambda)$ the eigenvector of the operator A, which corresponds to its eigenvalue $\lambda\in C$.

We consider system of evolution equations

$$\frac{d^2U_i(t)}{dt^2}+\left[\sum_{j=1}^{n}a_{ij}(A)\frac{d}{dt}+b_{ij}(B)\right]U_j(t)=f_i(t), \tag{1}$$

satisfies homogeneous nonlocal - integral conditions

$$p_i(A)U_i(t)\big|_{t=0}+q_i(A)U_i(t)\big|_{t=0}+\int_0^T U_i(t)dt=0, \tag{2}$$

$$p_i(A)\frac{dU_i}{dt}\bigg|_{t=0}+q_i(A)\frac{dU_i}{dt}\bigg|_{t=0}+\int_0^T tU_i(t)dt=0, \tag{3}$$

where $T>0$, $U_i:(0,T)\to H$ is an unknown vector-function, $p_i(\lambda)$, $q_i(\lambda)$, $i=\{1,2\}$, are given polynomials, $a(A)_{ij}, b_{ij}(B)$ are an abstract operators with entire symbols $a_{ij}(\lambda)\neq const$, $b_{ij}(\lambda)\neq const$, $\lambda\in C$, $f_i(t):(0,T)\to H$ is a given vector – function.

Definition. We shall say that for arbitrary $t\in(0,T), T>0$, the vector $f(t)$ from H belongs to $N_F(R,H,\Lambda^*)$, if on $\Lambda\subseteq C$ there exist a measure $\mu_f(\lambda)$ and analytical in t linear operator $F_f(t,\lambda):H\to H$ such that $f(t)$ can be represented in the form of Stieltjes integral

$$f(t)=\int_\Lambda F_f(t,\lambda)x(\lambda)d\mu(\lambda). \tag{5}$$

Theorem. Let be problem (1) – (3), $f(t)$ belong $N_F(R, H, \Lambda^*)$, and $f(t)$ can be represented in the form (5). Then the formula

$$U(t) = \int_{\Lambda^*} F_f\left(\frac{d}{dt}, \lambda\right)\{P(t, \mu, \lambda) x(\lambda)\}\Big|_{\nu=0} d\mu_f(\lambda)$$

defines formal solution of the problem (1) – (3), where $P(t, \mu, \lambda)$ is a solution of the equations $\phi\left(\frac{d}{dt}, \lambda\right) P(t, \mu, \lambda) = \exp[\lambda t]$, satisfies the conditions $\left.\frac{d^k P}{dt^k}\right|_{t=0} = 0, k = 0,1$.

This result continues the research of work [1, 2, 3].

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НЕЛОКАЛЬНА ЗАДАЧА З ІНТЕГРАЛЬНИМИ НЕОДНОРІДНОЇ УМОВАМИ ДЛЯ СИСТЕМИ ЕВОЛЮЦІЙНИХ РІВНЯНЬ ДРУГОГО ПОРЯДКУ

За допомогою диференціально-символьного методу побудовано розв'язок нелокальної задачі з інтегро-диференціальними умовами для системи операторних еволюційних рівнянь другого порядку.