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PROBLEM WITH INTEGRAL CONDITIONS FOR SYSTEM OF PARTIAL DIFFERENTIAL EQUATIONS

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Let K_L be a class of quasi-polynomials in the form $\phi(x)=\sum_{i=1}^{n}Q_i(x)\exp[\alpha_i x]$, where $Q_i(x)$ are given polynomials, $\alpha_i\in L\subseteq R$, $\alpha_l\neq\alpha_k$ for $l\neq k$. Each quasi-polynomial $\phi(x)$ defines a differential operator $\phi\left(\frac{\partial}{\partial\lambda}\right)\Phi(\lambda)\Big|_{\lambda=0}=$ $=\sum_{i=1}^{n}Q_i\left(\frac{\partial}{\partial\lambda}\right)\Phi(\lambda)\Big|_{\alpha_i}$ of finite order in the class of certain function $\Phi(\lambda)$.

In the strip $\Omega=\{(t,x)\in R^2: t\in(0,T), x\in R\}$ we consider system of equations

$$\frac{\partial U_i}{\partial t}+\sum_{j=1}^{n}a_{ij}\left(\frac{\partial}{\partial x}\right)U_j(t,x)=0,\qquad i=1,\dots,n, \tag{1}$$

satisfies conditions

$$U_i(t,x)\big|_{t=0}+U_i(t,x)\big|_{t=T}+\int_0^T U_i(t,x)dt=\phi_i(x), \tag{2}$$

where $a_{ij}\left(\frac{\partial}{\partial x}\right)$ are differentia expressions, which analytical symbols $a_{ij}(\lambda)$.

Let be $\eta(\lambda)=\int_0^T W^{(n-1)}(t,\lambda)dt$ is a certain function, $W(t,\lambda)$ is a solution of the equation $L\left(\frac{d}{dt},\lambda\right)W(t,\lambda)\equiv 0$, satisfies conditions $W^{(n-1)}(t,\lambda)\big|_{t=0}=1$, $W^{(n-2)}(t,\lambda)\big|_{t=0}=0,\dots,W(t,\lambda)\big|_{t=0}=0$.

Denote be

$$P=\{\lambda\in C:\eta(\lambda)=0\}. \tag{3}$$

Theorem. *Let* $\phi_i(x) \in K_M, i = 1,...,n$, *then the class* $K_{M\backslash P}$ *exist and unique solution of the problem (1), (2), where* P *is set (3), can be represented in the form*

$$U_i(t,x) = \sum_{i=1}^{n} \phi_i\left(\frac{\partial}{\partial\lambda}\right)\left\{\frac{1}{\eta(\lambda)} \overset{\sim T}{l}\left(\frac{d}{dt},\lambda\right) W(t,\lambda)\exp[\lambda x]\right\}\Bigg|_{\lambda=0},$$

where $\overset{\sim T}{l}\left(\frac{d}{dt},\lambda\right)$ *is transpose of a matrix.*

Be means of the differential-symbol method [1] we construction the solution of the problem (1), (2). This problem is a continues works [2,3,4,5].

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ЗАДАЧА З ІНТЕГРАЛЬНИМИ УМОВАМИ ДЛЯ СИСТЕМИ ДИФЕРЕНЦІАЛЬНИХ РІВНЯНЬ ІЗ ЧАСТИННИМИ ПОХІДНИМИ

За допомогою диференціально-символьного методу побудовано розв'язок задачі з інтегральними умовами для системи диференціальних рівнянь із частинними похідними першого порядку за виділеною змінною в класі квазімногочленів і доведено його єдиність.