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PRIME, DIFFERENTIALLY PRIME AND QUASI-PRIME SUBSEMIMODULES

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A set R together with two associative binary operations, called addition (+) and multiplication ($\cdot$), is called a *semiring* provided addition is commutative and multiplication distributes over addition both from the left and from the right. A semiring R is called *commutative* if multiplication is commutative.

A *semiring derivation* is defined in [1] as an additive map $\delta: R \to R$ satisfying $\delta(ab) = \delta(a)b + a\delta(b)$ for any $a, b \in R$. A semiring R equipped with a derivation δ is called *differential* with respect to the derivation δ, or δ-semiring. In [2], the authors investigated some simple properties of semiring derivations and differential semirings. The objective of this paper is to investigate some interrelations between prime, quasi-prime and differentially prime subsemimodules of differential semimodules.

In what follows let R be a δ-semiring, not necessarily commutative, and let M be a left semimodule over the semiring R. A map $d: M \to M$ is called a *derivation* of the semimodule M, associated with the semiring derivation $\delta: R \to R$ if $d(m+n) = d(m) + d(n)$ and $d(rm) = \delta(r)m + rd(m)$ for any $m, n \in M$, $r \in R$. A left R-semimodule M together with a derivation $d: M \to M$ is called a *differential semimodule*.

A subsemimodule N of the R-semimodule M is called *differential* if $d(N) \subseteq N$. For a subset $X \subset M$ a *differential* of X is defined as a set $X_{\#} = \left\{ x \in M \,\middle|\, x^{(n)} \in X \quad \forall n \in \mathbb{N} \cup \{0\} \right\}$, where $x^{(n)}$ denotes the n-th derivative of $x \in M$.

A non-empty subset T of the semimodule M is called an Sm*-system* of M if for every $s \in S$ and $t \in T$ there exists $r \in R$ such that $srt \in T$. A differential subsemimodule N of M will be called *quasi-prime* if it is maximal among differential subsemimodules of M disjoint from some Sm-system of M. Every prime subsemimodule, which is also differential, is quasi-prime, so is maximal subsemimodule.

Theorem. *For every differential subsemimodule* N *of the differential semimodule* M *with the ascending chain condition on differential subsemimodules the following are equivalent:*

1. N *is a differentially prime subsemimodule;*
2. N *is a quasi-prime subsemimodule;*
3. $N = P_{\#}$ *for some prime subsemimodule* P *of* M.

1. *Golan J.* Semirings and their Applications. – Springer Netherlands, 1999 – 382 p.
2. *Chandramouleeswaran M., Thiruveni V.* On derivations of semirings // Advances in Algebra. – 2010. – Vol. 1, No. 1. – P. 123–131.

ПЕРВИННІ, ДИФЕРЕНЦІАЛЬНО-ПЕРВИННІ ТА КВАЗІПЕРВИННІ ПІДНАПІВМОДУЛІ

У роботі досліджено диференціальні напівмодулі та їх диференціальні піднапівмодулі. Досліджено взаємозв'язки між квазіпервинними та диференціально-первинними піднапівмодулями в деяких напівмодулях.