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THE PROBLEM WITH DISTRIBUTED DATA FOR PARTIAL DIFFERENTIAL EQUATIONS

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In domain $\mathcal{D}^p := \left\{(t,x) : t > 0, x \in \mathbb{R}^p\right\}$ we investigate the conditions of existence of almost periodic for x with given spectrum $\mathcal{M} = \left\{\mu_k \in \mathbb{R}^p : \lim_{|k|\to\infty} |\mu_k| = \infty,\right.$ $\left. k \in \mathbb{Z}^p\right\}$ solution of the following problem:

$$L\left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}\right)[u] := \left(\frac{\partial^n}{\partial t^n} + \sum_{j=0}^{n-1} A_j\left(\frac{\partial}{\partial x}\right)\frac{\partial^j}{\partial t^j}\right)u(t,x) = 0, \quad (t,x) \in \mathcal{D}^p, \qquad (1)$$

$$U_j[u] := \int_{t_0}^{\infty} \exp(-b_j t) u(t,x) dt = \varphi_j(x), \quad x \in \mathbb{R}^p, \qquad (2)$$

where $\partial/\partial x = (\partial/\partial x_1, \ldots, \partial/\partial x_p)$, $A_j(\eta) = \sum_{|s|\le N_j} a_{js}\eta^s$, $\eta^s = \eta_1^{s_1} \cdots \eta_p^{s_p}$, $\eta \in \mathbb{R}^p$, $a_{js} \in \mathbb{C}$, $s \in \mathbb{Z}_+^p$; $b_j > 0, j = 1,\ldots,n$, $b_q \ne b_l, q \ne l$; $\varphi_j(x), j = 1,\ldots,n,$ are functions almost periodic for x with given spectrum $\mathcal{M}$.

Problems like (1), (2) arise in mathematical modelling of some heat conduction processes, moisture transfer, in problems of mathematical biology, of the long-term weather forecasting, etc. Problems with integral conditions for a time variable for evolution equations are, in general, ill-posed and their solvability in many cases are related to the problem of small denominators [1].

We assume that characteristic polynomial for equation (1) $L(\lambda, i\eta) = 0$, $\lambda \in \mathbb{R}$, $\eta \in \mathbb{R}^p \setminus \{0\}$, has only simple λ-roots for all η. By $\lambda_{jk} := \lambda_j(\mu_k)$ we denote the roots of $L(\lambda, i\mu_k) = 0$: $\left|\lambda_{jk}\right| \le C_1\left(1 + |\mu_k|\right)^\gamma$, $\gamma = \max_{1\le j<n}\left\{N_j/(n-j)\right\}$, $C_1 > 0$, $\Lambda_{\min,k} := \min_{1\le j\le n} \operatorname{Re}\lambda_{jk}$, $\Lambda_{\max,k} := \max_{1\le j\le n} \operatorname{Re}\lambda_{jk}$. By $f_{qk}(t)$, $q = 1,\ldots,n,$

we denote the normal (at point $t=0$) fundamental system of solutions of equation $L(d/dt, i\mu_k)=0;$ $\Delta(\mu_k,t_0):=\det\left\|\int_{t_0}^{\infty}\exp(-b_j t)f_{qk}(t)dt\right\|_{j,q=1}^{n}$.

Lemma 1. If $\Lambda_{\max,k}<\min_{1\le j\le n}\{b_j\}$ for all $k\in\mathbb{Z}^p$ then $\Delta(\mu_k,t_0)\neq 0$.

Under assumption of lemma the formal solution of the problem (1), (2) expressed by series

$$u(t,x)=\sum_{k\in\mathbb{Z}^p}\left(\sum_{j,q=1}^{n}\frac{\Delta_{jq}(\mu_k,t_0)}{\Delta(\mu_k,t_0)}\varphi_{jk}f_{qk}(t)\right)\exp(i\mu_k,x), \quad (3)$$

where $(\mu_k,x)=\left(\mu_{k_1},x_1+\cdots+\mu_{k_p}x_p\right)$, $\Delta_{jq}(\mu_k,t_0)$ is cofactor of the element in j-th row and q-th column in $\Delta(\mu_k,t_0)$, φ_{jk} is Fourier coefficient of $\varphi_j(x)$.

By $W_{\mathcal{M}}^{\alpha,\beta_k}$, $\alpha\in\mathbb{R}$, we denote a completion of space of finite sums $\sum v_k\exp(i\mu_k,x)$, $v_k\in\mathbb{C}$, with respect to the norm $\left\|v;W_{\mathcal{M}}^{\alpha,\beta_k}\right\|=\left(\sum_{k\in\mathbb{Z}^p}|v_k|^2\times\right.$ $\left.\times\left(1+|\mu_k|\right)^{2\alpha}\exp(2\beta_k)\right)^{\frac{1}{2}}$, where β_k is some sequence such that $\lim_{|k|\to\infty}\beta_k=\infty$.

Theorem 1. If $\Lambda_{\max,k}<0$ for all $k\in\mathbb{Z}^p$ and $\varphi_j\in W_{\mathcal{M}}^{\alpha,\beta_k}$, $j=1,\dots,n,$ then exists the unique solution $u(t,\cdot)$ of the problem (1), (2) defined by formula (3) and for every fixed $t>0$ it belongs to the space $W_{\mathcal{M}}^{\alpha-n\gamma(3n+1)/2,\beta_k+\omega_k(t)}$, where $\omega_k(t)=n(\Lambda_{\min,k}-\Lambda_{\max,k})t_0$ if $t\le t_0$ and $\omega_k(t)=-n\Lambda_{\max,k}(t-t_0)$ if $t>t_0$.

1. *Ptashnyk B. Y.* Ill-posed boundary value problems for partial differential equations. – Kyiv: Nauk. Dumka, 1984. – 264 p.

ЗАДАЧА З РОЗПОДІЛЕНИМИ ДАНИМИ ДЛЯ РІВНЯНЬ ІЗ ЧАСТИННИМИ ПОХІДНИМИ

Встановлено умови існування єдиного розв'язку задачі з інтегральними умовами для лінійних рівнянь із частинними похідними та виділено випадки у яких питання існування розв'язку задачі не пов'язане з проблемою малих знаменників.