

NONLOCAL PROBLEM WITH INTEGRAL CONDITION FOR SYSTEM OF EVOLUTION EQUATIONS OF SECOND ORDER

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Let H be Banach space, let A be linear operator acting in it $A: H \rightarrow H$, and for this operator arbitrary powers $A^n, n = 2, 3, \dots$ be also defined in H . Denote by $x(\lambda)$ the eigenvector of the operator A , which corresponds to its eigenvalue $\lambda \in C$.

We consider nonlocal problem for differential-operator equations

$$\frac{d^2 U_i(t)}{dt^2} + \left[\sum_{j=1}^n a_{ij} A \frac{d}{dt} + b_{ij}(B) \right] U_j(t) = 0,$$

satisfies non homogeneous integral conditions

$$p_i(A)U_i(t)\Big|_{t=0} + q_i(A)U_i(t)\Big|_{t=0} + \int_0^T U_i(t)dt = \varphi_i,$$

$$p_i(A) \frac{dU_i}{dt} \Big|_{t=0} + q_i(A) \frac{dU_i}{dt} \Big|_{t=0} + \int_0^T tU_i(t)dt = \psi_i,$$

where $T > 0$, $U_i: (0, T) \rightarrow H$ is an unknown vector-function, $p_i(\lambda), q_i(\lambda), i = \{1, 2\}$, are given polynomials, $a(A)_{ij}, b_{ij}(A)$ are an abstract operators with entire symbols $a_{ij}(\lambda) \neq const, b_{ij}(\lambda) \neq const, \lambda \in C$.

Definition. We shall say that vector φ from H belongs $L \subseteq H$, if on $\Lambda \subseteq C$ there exist depending on φ linear operator $R_\varphi(\lambda): H \rightarrow H, \lambda \in \Lambda$, and measure $\mu_\varphi(\lambda)$ such that

$$\varphi = \int_{\Lambda} R_{\varphi}(\lambda)x(\lambda)d\mu_{\varphi}(\lambda).$$

Theorem. *Let in the nonlocal conditions (2), (3) the vectors φ_1, φ_2 belong to L , i.e. φ_1, φ_2 can be represented in the form $\varphi = \int_{\Lambda} R_{\varphi}(\lambda)x(\lambda)d\mu_{\varphi}(\lambda)$, $i \in \{1,2\}$, where $\lambda \in \Lambda \setminus P$, where P is set zeros function $\Delta(\lambda)$. Then the formula*

$$U(t) = \sum_{m=0}^1 \int_{\Lambda} R_{\varphi_m}(\lambda)\{M_m(t, \lambda)x(\lambda)\}d\mu_{\varphi_m}(\lambda).$$

defines a formal solution of the problem (1), (2).

We construct a solution of the problem (1), (2), (3) with the use of the differential-symbol method [1, 2].

1. *Kalenyuk P.I., Nytrebych Z.M.* Generalized Scheme of Separation of Variables. Differential-Symbol Method – Publishing House of Lviv Polytechnic Natyonal University, 2002. – 292 p. (in Ukrainian).
2. *Kalenyuk P.I., Kuduk G., Kohut I.V., Nytrebych Z.M.* Problem with integraf conditions for differential operator equation // J. Math. Sci. – 2015. 208, No. 3. – P.267-276.

НЕЛОКАЛЬНА ЗАДАЧА З ІНТЕГРАЛЬНИМИ УМОВАМИ ДЛЯ СИСТЕМИ ЕВОЛЮЦІЙНИХ РІВНЯНЬ ДРУГОГО ПОРЯДКУ

За допомогою диференціально-символьного методу побудовано розв'язок нелокальної задачі з інтегро-диференціальними умовами для системи операторних еволюційних рівнянь другого порядку.