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NONLOCAL PROBLEM WITH INTEGRAL CONDITION FOR BICALORICAN DIFFERENTIAL EQUATIONS OF SECOND ORDER

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Let $H(([T_1,T_2]\times[T_3,T_4])\times R)$ be a class of certain functions, K_L be a class of quasi-polynomials in the form $\varphi(x)=\sum_{i=1}^{n} R_i(x)\exp[\alpha_i x]$, where $R_i(x)$ are given polynomials, $\alpha_i \in L \subseteq C$, for $\alpha_l \neq \alpha_k$, for $l \neq k$, for $k \in N, l \in N$.

In the strip $\Omega=\{(t,x)\in R^2 : t\in(0,T), x\in R\}$ we consider problem

$$\left[\frac{\partial}{\partial t}-a\left(\frac{\partial}{\partial x}\right)\right]^2 U(t,x)=0, \qquad (1)$$

satisfies conditions

$$\int_{T_1}^{T_2} U(t,x)dt+\int_{T_3}^{T_4} U(t,x)dt=\varphi_1(x),\cdot \qquad (2)$$

$$P\left(\frac{\partial}{\partial x}\right)\frac{\partial U(t,x)}{\partial t}\bigg|_{t=T_1}+Q\left(\frac{\partial}{\partial x}\right)\frac{\partial U(t,x)}{\partial t}\bigg|_{t=T_2}+\int_{T_1}^{T_2} tU(t,x)dt=\varphi_2(x),\cdot \qquad (3)$$

where $0<T_1<T_2<T_3<T_4$, $a\left(\frac{\partial}{\partial x}\right)$ are differential expression, with analytical symbol $a(\lambda)\neq const$, $P\left(\frac{\partial}{\partial x}\right), Q\left(\frac{\partial}{\partial x}\right)$ are given differential polynomials, with analytical functions $P(\lambda), Q(\lambda)$.

Теорема. *Let $\varphi_1(x),\varphi_2(x)\in K_L$, then the class $K_{L\setminus P}$ exist and unique solution of the problem (1), (2), where P is set (3), can be represented in the form*

$$U(t,x)=\varphi_1\left(\frac{\partial}{\partial\lambda}\right)\{M(t,\lambda)\exp[\lambda x]\}\bigg|_{\lambda=0}+\varphi_2\left(\frac{\partial}{\partial\lambda}\right)\{M(t,\lambda)\exp[\lambda x]\}\bigg|_{\lambda=0},$$

where $M(t,\lambda)$ is a solution of the equations

$$\left[\frac{d}{dt}-a(\lambda)\right]^2 M_m(t,\lambda)=0,$$

satisfies nonlocal-integral conditions

$$\int_{T_1}^{T_1} M_1(t,\lambda)dt+\int_{T_3}^{T_4} M_1(t,\lambda)dt=1,$$

$$P(\lambda)\frac{dM_1(t,\lambda)}{dt}\bigg|_{t=T_2}+Q(\lambda)\frac{dM_1(t,\lambda)}{dt}\bigg|_{t=T_2}+\int_{T_3}^{T_4} tM_1(t,x)dt=0,$$

$$\int_{T_1}^{T_1} M_2(t,\lambda)dt+\int_{T_3}^{T_4} M_2(t,\lambda)dt=0,$$

$$P(\lambda)\frac{dM_2(t,\lambda)}{dt}\bigg|_{t=T_2}+Q(\lambda)\frac{dM_2(t,\lambda)}{dt}\bigg|_{t=T_2}+\int_{T_3}^{T_4} tM_2(t,x)dt=1.$$

Solution of the problem (1), (2), according to the differential-symbol method [1] exists and unique in the class of quasi-polynomials. This result continues the research of work [2, 3].

1. *Kalenyuk P.I. Nytrebych Z.M.* Generalized Scheme of Separation of Variables. Differential-Symbol Method. – Publishing House of Lviv Polytechnic Natyonaly University, 2002. – 292 p. (in Ukrainian).
2. *Kalenyuk P.I. Nytrebych Z. M. Kohut I. V. Kuduk G.* Problem for nonhomogeneous second order evolution equation with homogeneous integral conditions, Math. Methods and Phys.- Mech. Polia. – 2015. – Vol. 58, No 1. – P. 7–19.
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НЕЛОКАЛЬНА ЗАДАЧА З ІНТЕГРАЛЬНИМИ УМОВАМИ ДЛЯ ПОЛІКАЛОРИЧНОГО РІВНЯНЬ ДРУГОГО ПОРЯДКУ

За допомогою диференціально-символьного методу побудовано розв'язок задачі з інтегро-диференціальними умовами для полікалоричного рівняння другого порядку.