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# RECONSTRUCTION OF ENERGY-DEPENDENT STURM–LIOUVILLE EQUATIONS

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The main object of our research is an energy-dependent Sturm–Liouville equation

$$-y''+qy+2\lambda py=\lambda^2 y \tag{1}$$

on (0,1) under some boundary conditions. Here $p$ is a real-valued function from $L_{2,\mathbb{R}}(0,1)$, $q=r'$ with $r\in L_{2,\mathbb{R}}(0,1)$ H (i.e. $q$ is a real-valued distribution from the Sobolev space $W_{2,\mathbb{R}}^{-1}(0,1)$), and $\lambda$ is a spectral parameter. We consider equation (1) under two types of boundary conditions: the Dirichlet ones

$$y(0)=y(1)=0 \tag{2}$$

and the mixed ones

$$y(0)=y^{[1]}(1)=0\,, \tag{3}$$

where $y^{[1]}:=y'-ry$ is a quasi-derivative of $y$.

For an eigenvalue $\lambda_n$ of the problem (1), (2), denote by $y_n$ the corresponding eigenfunction normalized by the initial conditions $y_n(0)=0$ and $y_n^{[1]}(0)=\lambda_n$. Then the quantity $\alpha_n:=2\int_0^1 y_n^2(t)dt-\frac{2}{\lambda_n}\int_0^1 p(t)y_n^2(t)dt$ is called the *norming constant* corresponding to the eigenvalue $\lambda_n$.

Denote by $A$ the operator acting as $Ay=\ell(y):=-y''+qy$ on the domain $\operatorname{dom} A:=\{y\in\operatorname{dom}\ell \mid y(0)=y^{[1]}(1)=0\}$. We assume that the following holds

(A) the operator $A$ is positive.

Under (A) the spectra of the problems (1), (2) and (1), (3) form an element of the set *SD1* which consists of all pairs $(\boldsymbol{\lambda},\boldsymbol{\mu})$ of increasing sequences $\boldsymbol{\lambda}=(\lambda_n)_{n\in\mathbb{Z}^*}$, $\mathbb{Z}^*:=\mathbb{Z}\setminus\{0\}$, and $\boldsymbol{\mu}=(\mu_n)_{n\in\mathbb{Z}}$ of real numbers satisfying the following conditions:

(i) asymptotics: $\lambda_n = \pi n + h + \tilde{\lambda}_n$, $\mu_n = \pi(n-1/2) + h + \tilde{\mu}_n$, with some $h \in R$ and $\ell_2$-sequences $(\tilde{\lambda}_n)$ and $(\tilde{\mu}_n)$;

(ii) almost interlacing: $\mu_k < \lambda_k < \mu_{k+1}$ for $k \in Z^*$.

Denote by $\mathbf{sd}(p,r)$ the set of all pairs $(\boldsymbol{\lambda}, \alpha)$, where $\boldsymbol{\lambda}$ is an eigenvalue of (1), (2) and $\alpha$ the corresponding norming constant. Under assumption (A) $\mathbf{sd}(p,r)$ forms an element of the set *SD2* which is the family of all sets $\{(\lambda_n, \alpha_n)\}_{n \in Z^*}$ consisting of pairs $(\lambda_n, \alpha_n)$ of real numbers satisfying the following properties:

(i) $\lambda_n$ are nonzero, strictly increasing with $n \in Z^*$, and have the representation $\lambda_n = \pi n + h + \tilde{\lambda}_n$ for some $h \in R$ and $\ell_2$-sequence $(\tilde{\lambda}_n)$;

(ii) $\alpha_n > 0$ for all $n \in Z^*$ and the numbers $\tilde{\alpha}_n := \alpha_n - 1$ form an $\ell_2$-sequence.

Our main results are the following theorems.

**Theorem 1.** ([2]) *For every pair* $(\boldsymbol{\lambda}, \boldsymbol{\mu})$ *from SD1, there exist unique real-valued* $p, r \in L_2(0,1)$ *such that* $\boldsymbol{\mu}$ *and* $\boldsymbol{\lambda}$ *are respectively the spectra of problems (1), (2) and (1), (3).*

**Theorem 2.** ([1]) *For every set* **sd** *from SD2, there exist unique real-valued* $p, r \in L_2(0,1)$ *such that* $\mathbf{sd} = \mathbf{sd}(p,r)$.

The reconstructing algorithms for both inverse problems will be formulated in the talk.


1. *Hryniv R., Pronska N.* Inverse spectral problems for energy-dependent Sturm–Liouville equations // Inverse Problems. – 2012. – 28 (8): 085008 – 21 pp.
2. *Pronska N.* Reconstruction of energy-dependent Sturm-Liouville equations from two spectra // Integral Equations and Operator Theory. – 2013. –76 (3) – P. 403–419.


## ВІДНОВЛЕННЯ ЕНЕРГОЗАЛЕЖНИХ РІВНЯНЬ ШТУРМА-ЛІУВІЛЛЯ

*Ми досліджуємо відновлення енергозалежних рівнянь Штурма–Ліувілля за двома спектрами та за спектром і набором відповідно визначених нормівних множників. Встановлено існування, єдиність та алгоритми відновлення.*