Конференція молодих учених «Підстригачівські читання – 2017», 23–25 травня 2017 р., Львів

УДК 532.595

STEADY-STATE RESONANT DAMPED SLOSHING IN UPRIGHT CIRCULAR TANK

Ihor Raynovskyy

Institute of Mathematics of NASU, ihor.raynovskyy@gmail.com

By using a nonlinear Narimanov-Moiseev modal theory [1], we study the linear damping effect on the steady-state resonant sloshing in an upright circular tank. The tank performs periodic sway/surge/roll/pitch motions with the forcing frequency σ close to the lowest natural sloshing frequency σ_{11}. The surface tension is neglected that suggests, as we show, the relatively large tank radius ($r_0 \geq 5$ cm) .The damping terms accounts for the logarithmic decrements of the natural sloshing modes due to frictional (laminar boundary layer) effect on the wetted tank surface as well as the viscous bulk dissipation. The damping coefficients are theoretically evaluated by using Miles' [2] asymptotic formula in terms of the Galilei number. Numerical analysis shows that the damping coefficients are of the same asymptotic order in the adopted theory; they can be neglected for higher modes and, totally, in the theory as $r_0 \geq 20$ cm .This means that the presented studies are of practical interest for laboratory tanks (e.g., bioreactors) with $5 \text{ cm} \leq r_0 \leq 20$ cm .

The analytical asymptotic steady-state solutions of the Narimanov-Moiseev modal equations are derived and their stability is analysed by combining the linear Lyapunov method and the multi-timing technique. We prove that, within to higher-order quantities, the steady-state solutions are the same as for those appearing due to either longitudinal or elliptic or rotary harmonic tank excitations. The analytical steady-state results can be numerically treated in terms of the three-dimensional $(\sigma / \sigma_{11}, A, B)$ response curves, where A and B are the lowest-order amplitude parameters, which correspond to the primary-excited lowest (degenerating) sloshing modes.

The typical damped *vs.* undamped response curves for the longitudinal forcing type are presented in fig.1 when planar and swirling steady-state wave types are theoretically and experimentally confirmed. Comparing (a, undamped) and (b, damped) cases shows that the linear damping can dramatically affect swirling so that an arc-like response curve $P_1H_1H_2P_2$ appears instead of the branches with infinitely-located points, which are demonstrated in the panel (a). The results on the response curves are generalized for elliptic and rotary forcing

types. The theoretical expectations on the steady-state wave elevation and hydrodynamic forces are supported by experiments.

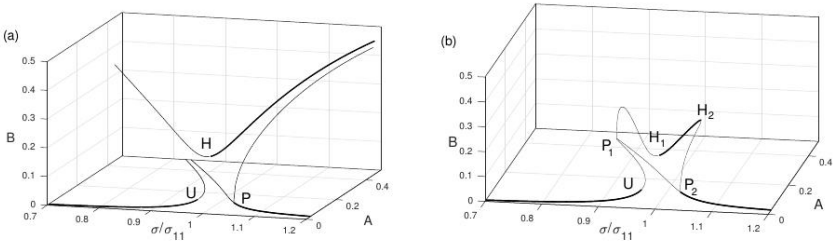

Figure 1:The undamped (a) and damped (b) response curves of the steady-state sloshing for a longitudinal forcing type. The liquid depth-to-r_0 ratio is 1.5, the nondimensional forcing amplitude is equal 0.01, the damping coefficient for the primary-excited mode in (b) equals to 0.02.The solid lines denote the stability. The response curves belonging to the $(\sigma/\sigma_{11}, A)$ plane correspond to the so-called planar wave regime but the three-dimensional curves imply swirling. There are no stable steady-state sloshing (irregular, chaotic waves are expected) in the frequency range between U and P in (a) (or P_2 in (b) respectively).

1. *Raynovskyy I. A., Timokha A. N.* Resonant liquid sloshing in an upright circular tank performing a periodic motion // Journal of Numerical and Applied Mathematics. – 2016. – No 2 (122). – P. 71-82.
2. *MilesJ. W., Henderson D. M.* A note on interior vs. boundary-layer damping of surface waves in a circular cylinder // Journal of Fluid Mechanics. – 1998. – Vol.364. –P. 319-323.

УСТАЛЕНІ РЕЗОНАНСНІ КОЛИВАННЯ РІДИНИ У ВЕРТИКАЛЬНОМУ КРУГОВОМУ КОНТЕЙНЕРІ ЗА НАЯВНОСТІ ДЕМПФУВАННЯ

Використовується нелінійна модальна теорія Наріманова-Моісєєва для дослідження усталених резонансних хвиль, зумовлених періодичними рухами баку з частотою, близькою до першої власної частоти коливання рідини, коли суттєвою є лінійна дисипація в системі. Теоретична оцінка останньої призводить до обмеження на радіус баку, яке є типовим для біореакторів. Побудовано аналітичний (періодичний) розв'язок задачі, досліджено його стійкість, прокласифіковано типи усталених хвиль. Теоретичні результати порівняно із відомими експериментальними. Проведено чисельний порівняльний аналіз між амплітудно-частотними характеристиками із та без врахування лінійної дисипації.