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WEAKLY NONLINEAR ANALYSIS OF ACTIVATOR-INHIBITOR SYSTEM WITH CUBIC NONLINEARITY IN THE SUPERDIFFUSIVE CASE

Zoya Prytula

Pidstryhach Institute for Applied Problems of Mechanics and Mathematics of the National Academy of Sciences of Ukraine, zoya777b@gmail.com

Experimental observations of spiral waves, spatial dissipative structures with complicated symmetries in many physical, chemical and biological media have made reaction-diffusion systems the subject of numerous investigations. Recently, many scientists noticed that the diffusion in real-life systems have got an anomalous character and cannot be described in terms of normal (Fickian) diffusion. These processes can be described using models with subdiffusion or superdiffusion [1-4].

The effect of superdiffusion on pattern formation and pattern selection in the Brusselator model is studied in [5]. The authors have performed a weakly nonlinear analysis and obtained a system of amplitude equations. The analysis of these equations allowed them to predict the parameter regimes where hexagons, stripes and their coexistence are expected.

In this work we study the generalized activator-inhibitor model with cubic nonlinearity, in which the classical Laplacian is replaced by a fractional operator that reflects the nonlocal behavior of superdiffusion.

For the considered model we have been found the spatially homogeneous, time independent solution and also studied its linear stability. As a result, we have obtained the Turing instability threshold A_{cr} and also the critical value of the wave number k_{cr}, which depends on superdiffusive exponent α.

By means of a weakly nonlinear analysis we have obtained a system of amplitude equations. The obtained amplitude equations form a basis for the analysis of pattern formation. In particular, we are interested in the steady states that describe hexagons and stripes in the generalized activator-inhibitor model with cubic nonlinearity.

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СЛАБКОНЕЛІНІЙНИЙ АНАЛІЗ СИСТЕМИ АКТИВАТОР-ІНГІБІТОРНОГО ТИПУ З КУБІЧНОЮ НЕЛІНІЙНІСЮ ДЛЯ ВИПАДКУ СУПЕРДИФУЗІЇ

Досліджено узагальнену модель типу активатор-інгібітор із кубічною нелінійністю, в якій класичний оператор Лапласа замінено дробовим аналогом. Дробовий оператор відображує нелокальну поведінку супердифузії. Для такої моделі знайдено просторово-однорідний стаціонарний розв'язок та вивчено його лінійну стійкість. Проведено слабконелінійний аналіз та отримано систему амплітудних рівнянь, яка є вихідною для прогнозування типу структур, а також режимів параметрів, для яких можуть існувати різноманітні структури.