**Конференція молодих учених «Підстригачівські читання – 2016»,
25–27 травня 2016 р., Львів**

УДК 512.552.1

ON KEIGHER SEMIRINGS

Ivanna Melnyk

Ivan Franko National University of Lviv, ivannamelnyk@yahoo.com

This work is devoted to investigation of commutative semirings with derivations in which the radical of each differential ideal is differential. A set R together with two associative binary operations, called addition (+) and multiplication ($\cdot$), is called a *semiring* provided addition is commutative and multiplication distributes over addition both from the left and from the right. A semiring R is called *commutative* if multiplication is commutative. The notion of a *semiring derivation* is defined in [1] as an additive map $\delta: R \to R$ satisfying. $\delta(ab) = \delta(a)b + a\delta(b)$ for any $a, b \in R$. A semiring R equipped with a derivation δ is called *differential* with respect to the derivation δ, or δ-semiring. In [2] the authors investigated some simple properties of semiring derivations and differential semirings. The objective of this paper is to extend some results on differential rings to differential semiring.

In what follows let R be a commutative δ-semiring. An ideal I of R is called *differential* if $\delta(a) \in I$ whenever $a \in I$. An ideal I of R is called *subtractive* (or *k*-ideal) if $a+b \in I$ and $a \in I$ follow that $b \in I$. A *prime ideal* of R is a proper ideal P of R in which $a \in P$ or $b \in P$ whenever $ab \in P$. A proper ideal I of R is said to be *maximal* (resp. *k-maximal*) if for any ideal (resp. *k*-ideal) J of R such that $I \subset J$, we have $J = R$. An ideal I of R is called *radical* if $a^n \in I$ follows $a \in I$ for $a \in R$, $n \in \mathbb{N} \cup \{0\}$.

Theorem 1. *Let R be a commutative differential semiring, and let $S \subset R$ be a multiplicatively closed subset of R ($0 \notin S$). If I is a radical differential ideal of R, maximal among radical differential ideals disjoint from S, then I is prime.*

Theorem 2. *Let R be a commutative differential semiring. If I is a radical differential ideal of R, then it is an intersection of prime differential ideals.*

For a subset $A \subset R$ a *differential* of A is defined as a set $A_\# = \left\{ a \in R \,\middle|\, a^{(n)} \in A \quad \forall n \in \mathbb{N} \cup \{0\} \right\}$, where $a^{(n)}$ denotes the n-th δ-derivative of a. A differential semiring R is called *a Keigher semiring* if $P_\#$ is a prime differential ideal for any prime ideal P of R.

Theorem 3. *For a commutative differential semiring* R *the following conditions are equivalent:*

1. R *is a Keigher semiring.*

2. If I *is a differential ideal, then so is* $\sqrt{I}$ *.*

3. If $S \subset R$ *is a multiplicatively closed subset of* R ($0 \notin S$) *and* I *is a differential ideal of* R *disjoint from* S *, then every differential ideal of* R *, which is maximal among differential ideals containing* I *and disjoint from* S *, is a prime ideal.*

4. If A *is any subset of* R *, then* $\{A\} = \sqrt{\{A\}}$ *.*

Every differentially trivial semiring is a Keigher semiring. Zero semiring $\{0\}$ is a Keigher semiring. Any differential semifield is a Keigher semiring. If R_1 is a Keigher semiring and $f : R_1 \to R_2$ is a differential semiring epimorphism then R_2 is a Keigher semiring.

Theorem 4. *Let* R_1 *, ...* R_n *be an arbitrary family of commutative differential semirings and let* $R = R_1 \times \ldots \times R_n$ *be their product. Then* R *is a Keigher semiring if and only if every* R_i *is a Keigher semiring.*

1. *Golan J.* Semirings and their Applications. – Springer Netherlands, 1999. – 382 p.
2. *Chandramouleeswaran M., Thiruveni V.* On derivations of semirings // Advances in Algebra. – 2010. – Vol. 1, No. 1. – P. 123–131.

ПРО НАПІВКІЛЬЦЯ КЕЙГЕРА

У роботі досліджено комутативні диференціальні напівкільця, в яких радикал кожного диференціального ідеалу є диференціальним ідеалом – напівкільця Кейгера. Встановлено еквівалентні умови, які визначають такі напівкільця, а також деякі їх властивості.