**Конференція молодих учених «Підстригачівські читання – 2016»,
25–27 травня 2016 р., Львів**

УДК 517.5

ON AN ORTHOGONAL TRIGONOMETRIC SCHAUDER BASIS AND THE LOCAL BESOV SPACES*

Nadiya Derevyanko

Institute of Mathematics, National Academy of Sciences of Ukraine, nadyaderevyanko@gmail.com

Let $1 \le p < \infty$. By L_p^* we denote the space of 2π-periodic measurable functions $f:\mathbb{R}\to\mathbb{R}$ with the finite norm $\|f\|_p = \left(\frac{1}{2\pi}\int_0^{2\pi} |f(t)|^p \, dt\right)^{1/p}$; L_∞^* is the space of 2π-periodic continuous functions with the finite norm $\|f\|_\infty = \sup_{t\in[0,2\pi)} |f(t)|$. Let $n\in\mathbb{N}$ and $N_n = 3\cdot 2^n$. By $T_n^s : L_p^* \to L_p^*$, we denote operators of the shifts $T_n^s f = f(\cdot - s\pi/N_n)$ for $s = 0,\dots,2N_n - 1$. We consider the following orthogonal trigonometric Schauder basis $\{t_\mu\}_{\mu=0}^{\infty}$ in the space L_p^* [1]: $t_k(\cdot) = T_1^k \varphi_1(\cdot)$, $k = 0,\dots,2N_1-1$, and for $n\in\mathbb{N}$, $s = 0,\dots,2N_n-1$ let $t_{2N_n+s}(\cdot) = T_n^s \psi_n(\cdot)$, where φ_1 is a scaling function of PMRA generated by de la Vallée Poussin means and ψ_n, $n\in\mathbb{N}$, are corresponding wavelets. A function $f\in L_p^*, 1\le p<\infty,$, can be given by the series $f(x) = \sum_{s=0}^{2N_1-1} \left(f*\varphi_1\right)(s\pi/N_1) T_1^s \varphi_1(x) + \sum_{n=1}^{\infty}\sum_{s=0}^{2N_n-1} \left(f*\psi_n\right)(s\pi/N_n) T_n^s \psi_n(x),$ where the convergence is understood in the metric of the space L_p^*.

Our aim is to describe the local Besov spaces $B_{p,\rho}^{\alpha}(x_0)$ [2] of periodic functions *f* by conditions on the coefficients in a series expansion of *f*.

Let $I\subset(0,2\pi)$ be an interval and $n\in\mathbb{N}$. For index $s = 0,\dots,2N_n-1$, the notation $s\in\kappa(I,n)$ means that the point $s\pi/N_n$ belongs to the interval *I*. For $1\le p\le\infty$ and the interval *I*, we define the following sequence

$$c_0(I,p) = \begin{cases} 2^{1/2-1/p}\left(\sum_{s\in\kappa(I,1)} |(f*\varphi_1)(s\pi/N_1)|^p\right)^{1/p}, & 1\le p<\infty, \\ 2^{1/2}\max_{s\in\kappa(I,1)} |(f*\varphi_1)(s\pi/N_1)|, & p=\infty, \end{cases}$$ and, for $n\in\mathbb{N}$,

* This talk is based on a joint research with Prof. Dr. Jürgen Prestin (Institut für Mathematik, Universität zu Lübeck) and Dr. Vitalii Myroniuk (Institute of Mathematics of NAS of Ukraine).

$$c_n(I,p)=\begin{cases}2^{n(1/2-1/p)}\left(\sum\limits_{s\in\kappa(I,n)}|(f*\psi_n)(s\pi/N_n)|^p\right)^{1/p}, & 1\le p<\infty,\\ 2^{n/2}\max\limits_{s\in\kappa(I,n)}|(f*\psi_n)(s\pi/N_n)|, & p=\infty.\end{cases}$$

Let ζ be an infinitely differentiable on $[0,2\pi)$ function, supported on *I*. By $\{d_n(I,\zeta,p)\}_{n=0}^{\infty}$, we denote $d_0(I,\zeta,p)=\begin{cases}2^{1/2-1/p}\left(\sum\limits_{s=0}^{2N_1-1}|\big((f\zeta)*\varphi_1\big)(s\pi/N_1)|^p\right)^{1/p}, & 1\le p<\infty,\\ 2^{1/2}\max\limits_{s=0,\ldots,2N_1-1}|\big((f\zeta)*\varphi_1\big)(s\pi/N_1)|, & p=\infty,\end{cases}$

and, for $n\in\mathbb{N}$, $d_n(I,\zeta,p)=\begin{cases}2^{n(1/2-1/p)}\left(\sum\limits_{s=0}^{2N_n-1}|\big((f\zeta)*\psi_n\big)(s\pi/N_n)|^p\right)^{1/p}, & 1\le p<\infty,\\ 2^{n/2}\max\limits_{s=0,\ldots,2N_n-1}|\big((f\zeta)*\psi_n\big)(s\pi/N_n)|, & p=\infty.\end{cases}$

Let us formulate our main result.

Theorem. *Let* $1\le p\le\infty$, $f\in L_p^*$, $x_0\in(0,2\pi)$, $0<\alpha<1$, $0<\rho\le\infty$. *The following statements are equivalent:(a)* $f\in B_{p,\rho}^{\alpha}(x_0)$ *; (b) There exists an interval* $I\subset(0,2\pi)$ *, centered at* x_0 *, such that the norm* $\left(\sum\limits_{n=0}^{\infty}\left(2^{n\alpha}|c_n(I,p)|\right)^{\rho}\right)^{1/\rho}$, $1\le\rho<\infty$, $\sup\limits_{n\in N_0}2^{n\alpha}|c_n(I,p)|$, $\rho=\infty$, *is finite ;(c)* $I\subset(0,2\pi)$, *centered at* x_0 *, such that for every infinitely differentiable on* $[0,2\pi)$ *function* ζ *, supported on I (and extended as a* 2π*-periodic function), the norm* $\left(\sum\limits_{n=0}^{\infty}\left(2^{n\alpha}|d_n(I,\zeta,p)|\right)^{\rho}\right)^{1/\rho}$, $1\le\rho<\infty$, $\sup\limits_{n\in N_0}2^{n\alpha}|d_n(I,\zeta,p)|$, $\rho=\infty$, *is finite.*

1. *Prestin J., Selig K. K.* On a constructive representation of an orthogonal trigonometric Schauder basis for $C_{2\pi}$ // Oper. Theory: Adv. Appl. – 2001. – **121**. – P. 402-425.
2. *Mhaskar H. N., Prestin J.* On local smoothness classes of periodic functions // J. Fourier Anal. Appl. – 2005. – **11**, № 3. – P. 353-373.

ОДИН ОРТОГОНАЛЬНИЙ ТРИГОНОМЕТРИЧНИЙ БАЗИС ШАУДЕРА І ЛОКАЛЬНІ ПРОСТОРИ БЄСОВА

Наведено нові результати стосовно проблеми опису локальних просторів Бєсова періодичних функцій однієї змінної за умов на коефіцієнти розвинень у ряди їхніх елементів за певним ортогональним тригонометричним базисом Шаудера.