

## ON EXACT SOLUTIONS OF SOME LINEAR AND NON-LINEAR P(1,4)-INVARIANT D'ALEMBERT EQUATIONS

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Consider the following differential equations:

$$\square_5 u = \lambda u, \lambda \in \mathbb{R}, \quad (1)$$

$$\square_5 u = \sin u, \quad (2)$$

$$\square_5 u = e^u, \quad (3)$$

$$\square_5 u = \sinh u, \quad (4)$$

where  $\square_5 = \frac{\partial^2}{\partial x_0^2} - \frac{\partial^2}{\partial x_1^2} - \frac{\partial^2}{\partial x_2^2} - \frac{\partial^2}{\partial x_3^2} - \frac{\partial^2}{\partial x_4^2}$  is the d'Alembert operator in the five-dimensional Minkowski space  $M(1,4)$ .

These equations are five-dimensional generalizations of ones, which have many applications in differential geometry, theory of nonlinear waves, theoretical and mathematical physics (see, e.g., [1-5]).

Equations (1)-(4) are invariant under the generalized Poincaré group  $P(1,4)$ .

By now, using the subgroup structure of the group  $P(1,4)$  as well as invariants of its nonconjugate subgroups, we have performed the symmetry reduction for the above mentioned equations and constructed some classes of exact solutions for them.

Among these solutions there are:

- solutions with elementary functions,
- solutions with special functions,
- solutions with two arbitrary functions.

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3. *Bhatnagar P. L.* Nonlinear Waves in One-Dimensional Dispersive Systems. – Oxford: Clarendon, 1979. – 142 pp.
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**ПРО ТОЧНІ РОЗВ'ЯЗКИ ДЕЯКИХ ЛІНІЙНИХ ТА НЕЛІНІЙНИХ  
P(1,4)-ІНВАРІАНТНИХ РІВНЯНЬ Д'АЛАМБЕРА**

*Використовуючи підгрупову структуру групи Пуанкаре  $P(1,4)$  проведено симетрійну редуцію деяких  $P(1,4)$ -інваріантних рівнянь Д'Аламбера та побудовано деякі класи їх точних розв'язків.*