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HP-ADAPTIVE FINITE ELEMENT APPROXIMATION FOR 1D BOUNDARY VALUE PROBLEMS

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This talk is devoted to *hp*-adaptive algorithm for finite element method (FEM) for one-dimensional convection-diffusion-reaction boundary value problems with self-adjoint operators. It is based on a combination of classical implicit and explicit a posteriori error estimators (AEE). The main focus is on the construction of refinement pattern selection procedure. Results of computational experiments are used to compare the developed adaptive scheme with algorithm based on *reference solution* [1].

Model problem

Let us consider the following boundary value problem: find function $u = u(x)$ such that

$$\begin{cases} -(\mu u')' + \beta u' + \sigma u = f \quad \text{in} \quad \Omega = (0, L), \\ (\mu u')\big|_{x=0} = \alpha[u(0) - \bar{u}_0], \quad -(\mu u')\big|_{x=L} = \gamma[u(L) - \bar{u}_L], \end{cases} \tag{1}$$

where $\alpha, \gamma \ge 0$, $\mu(x) \ge \mu_0 > 0$, $\sigma(x) \ge 0$ almost everywhere in Ω, $\mu, \beta, \sigma \in L^\infty(0, L)$, $f \in L^2(0, L)$. The variational problem, which corresponds to (1), is solved approximately by making use of FEM with high-order piecewise polynomial basis functions.

Adaptation algorithm

The described research follows previous investigation of adaptive FEM algorithm based on reference solution. Despite significant advantage in versatility compared to other types of algorithms, this approach has a significant drawback, namely the difficulty of theoretical investigation and justification of correctness. These deficiencies are the main factors to slow the development of commercial applications based on adaptive *hp*-FEM algorithms. Therefore it was decided to use well studied classical AEEs as a basis of the developed algorithm. The algorithm combines the use of explicit and implicit AEEs. Explicit AEE is used for the selection of finite elements, which need to be refined, and implicit to select the optimal element refinement patterns.

Numerical results

Let us consider problem (1) with following data:

$$\mu = 1, \beta = 0, \sigma = 10^5 e^x, f = 10^5, \alpha = \gamma = 10^8, \overline{u}_0 = \overline{u}_L = 0, L = 1. \qquad (2)$$

Table 1. Algorithm convergence history: acceptable level of relative error – 5%, maximum approximation polynomial degree – 9. Absolute error indicator is computed as weighted L^2-norm of residual of problem (1) equation.

iteration	D.O.F.	absolute error indicator	relative error indicator, %
0	*3*	*12579,37*	*6045,54*
1	*5*	*3998,07*	*1695,35*
… … …			
7	*34*	*25,53*	*10,19*
8	*39*	*24,94*	*9,95*
9	*43*	*7,59*	*3,03*

For data (2), algorithm based on reference solution terminates after 32 iterations (>3 times greater) with final count of D.O.F. – 437 (10 times greater). The data for 9 iterations are given in Table 1.

1. *Demkowicz L.* Computing with hp-adaptive finite elements. I. One- and two-dimensional elliptic and Maxwell problems. – Boca Raton: Chapman & Hall/CRC, 2007. – 398 p.
2. *Dorfler W., Heuveline V.* Convergence of an adaptive *hp* finite element strategy in one space dimension // Appl. Num. Math. – 2007. – 57, No 10. – P. 1108-1124.

АПРОКСИМАЦІЯ *HP*-АДАПТИВНОГО МЕТОДУ СКІНЧЕННИХ ЕЛЕМЕНТІВ ДЛЯ ОДНОВИМІРНИХ КРАЙОВИХ ЗАДАЧ

Запропоновано hp-адаптивний алгоритм методу скінченних елементів (МСЕ) для одновимірних крайових задач конвекції-дифузії-реакції із самоспряженими операторами. Алгоритм ґрунтується на комбінування явних та неявних апостеріорних оцінювачів похибок. Основну увагу зосереджено на побудові процедури вибору оптимального способу локальної перебудови сітки МСЕ. За результатами обчислювальних експериментів проведено порівняльний аналіз розробленого алгоритму та алгоритмму на основі контрольного розв'язку.