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VIBRATION OF ORTHOTROPIC CYLINDRICAL SHELL WITH A SET OF CUTOUTS OF ARBITRARY CONFIGURATION

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The problem on the steady state vibrations of the orthotropic cylindrical shell is considered. The shell has N cutouts of the arbitrary shape and location. The contours of the cutouts are the curves $L^{(j)}, j=\overline{1,N}$. The external boundaries of the shell are also of the arbitrary form, and its contours are the curves $L^{(0)}, L^{(N+1)}$. One can imagine this object that in terms of middle surface covers the multi-connected domain Ω, as the result of an arbitrary cutoff from the cylindrical shell that in terms of middle surface covers the singly connected domain Π of the canonical form. The curvilinear system of coordinates has been attached to this fictitiously extended domain classically. For the simplicity of the formulas it is taken that principle axes of the orthotropy coincide with the directions of the coordinate lines.

Such notations are used: $\mathbf{n}, \boldsymbol{\tau}$ − normal and tangential vector to the certain direction; E_i − Young's moduli; G_{12}, G_{13}, G_{23} − shear moduli of the material; ν_{12}, ν_{21} − Poisson coefficients; ρ − material density; k_1, k_2 − principle curvatures ; $R, l, 2h$ − radius, length, and thickness of the shell respectively; q_i, m_i − components of the external load; w — deflection; $u_{in}, u_{i\tau}$ − normal and tangential components of the displacements of the points in the middle surface; $\gamma_{in}, \gamma_{i\tau}$ − normal and tangential components of the rotation angles of the normal to the middle surface; Q_n − normal component of the shear force; M_n, N_n − normal components and M_τ, N_τ − tangential components of the moment and axial force.

Let us consider two different types of the harmonic in time boundary conditions with the frequency ω on all the contours $L^{(j)}, j=\overline{0,N+1}$:

a) Distributed components of the displacements are given

$$w^{(j)}=w_0^{(j)}(\alpha)e^{i\omega t},\quad u_n^{(j)}=u_{n0}^{(j)}(\alpha)e^{i\omega t},\quad \gamma_n^{(j)}=\gamma_{n0}^{(j)}(\alpha)e^{i\omega t},$$
$$u_\tau^{(j)}=u_{\tau 0}^{(j)}(\alpha)e^{i\omega t},\quad \gamma_\tau^{(j)}=\gamma_{\tau 0}^{(j)}(\alpha)e^{i\omega t}. \tag{1}$$

b) Distributed components of the forces are given:

$$Q_n^{(j)} = Q_{n0}^{(j)}(\alpha)e^{i\omega t}, \quad M_n^{(j)} = M_{n0}^{(j)}(\alpha)e^{i\omega t}, \quad N_n^{(j)} = N_{n0}^{(j)}(\alpha)e^{i\omega t},$$
$$N_\tau^{(j)} = N_{\tau 0}^{(j)}(\alpha)e^{i\omega t}, \quad M_\tau^{(j)} = M_{\tau 0}^{(j)}(\alpha)e^{i\omega t}. \qquad (2)$$

The system of key equations that take into account all inertial components and shear displacements can be expressed as [1]:

$$[\mathbf{L}]\{U\} = -\{P\}, \qquad (3)$$

$$\{U\} = \{u_1, u_2, w, \gamma_1, \gamma_2\}, \quad \{P\} = \{q_1, q_2, q_3, m_1, m_2\},$$

$$\mathbf{L}_{11} = B_1\frac{\partial^2}{\partial\alpha_1^2} + B_{12}\frac{\partial^2}{\partial\alpha_2^2} - k_1^2\Lambda_1 - 2h\rho\frac{\partial^2}{\partial t^2},$$

$$\mathbf{L}_{22} = B_{12}\frac{\partial^2}{\partial\alpha_1^2} + B_2\frac{\partial^2}{\partial\alpha_2^2} - k_2^2\Lambda_2 - 2h\rho\frac{\partial^2}{\partial t^2},$$

$$\mathbf{L}_{33} = \Lambda_1\frac{\partial^2}{\partial\alpha_1^2} + \Lambda_2\frac{\partial^2}{\partial\alpha_2^2} - \left[k_1 B_1\left(k_1 + \nu_{12}k_2\right) + k_2 B_2\left(k_2 + \nu_{21}k_1\right)\right] - 2h\rho\frac{\partial^2}{\partial t^2},$$

$$\mathbf{L}_{44} = D_1\frac{\partial^2}{\partial\alpha_1^2} + D_{12}\frac{\partial^2}{\partial\alpha_2^2} - \Lambda_1 - 2h^2\rho\frac{\partial^2}{\partial t^2}, \ \mathbf{L}_{34} = -\mathbf{L}_{43} = \Lambda_1\frac{\partial}{\partial\alpha_1},$$

$$\mathbf{L}_{55} = D_{12}\frac{\partial^2}{\partial\alpha_1^2} + D_2\frac{\partial^2}{\partial\alpha_2^2} - \Lambda_2 - 2h^2\rho\frac{\partial^2}{\partial t^2}, \mathbf{L}_{35} = -\mathbf{L}_{53} = \Lambda_2\frac{\partial}{\partial\alpha_2},$$

$$\mathbf{L}_{14} = \mathbf{L}_{41} = k_1\Lambda_1, \ \mathbf{L}_{25} = \mathbf{L}_{52} = k_2\Lambda_2, \ \mathbf{L}_{15} = \mathbf{L}_{51} = 0, \mathbf{L}_{24} = \mathbf{L}_{42} = 0,$$

$$\mathbf{L}_{12} = \left(B_1\nu_{12} + B_{12}\right)\frac{\partial^2}{\partial\alpha_1\partial\alpha_2}, \quad \mathbf{L}_{21} = \left(B_{12} + B_2\nu_{21}\right)\frac{\partial^2}{\partial\alpha_1\partial\alpha_2},$$

$$\mathbf{L}_{45} = \left(D_1\nu_{12} + D_{12}\right)\frac{\partial^2}{\partial\alpha_1\partial\alpha_2}, \quad \mathbf{L}_{54} = \left(D_{12} + D_2\nu_{21}\right)\frac{\partial^2}{\partial\alpha_1\partial\alpha_2},$$

$$\mathbf{L}_{13} = \left(k_1\Lambda_1 + B_1k_1 + B_1k_2\nu_{12}\right)\frac{\partial}{\partial\alpha_1}, \quad \mathbf{L}_{31} = -\left(k_1\Lambda_1 + B_1k_1 + B_2k_2\nu_{21}\right)\frac{\partial}{\partial\alpha_1},$$

$$\mathbf{L}_{23} = \left(k_2\Lambda_2 + B_2k_2 + B_2k_2\nu_{21}\right)\frac{\partial}{\partial\alpha_2}, \ \mathbf{L}_{32} = -\left(k_2\Lambda_2 + B_2k_2 + B_1k_1\nu_{12}\right)\frac{\partial}{\partial\alpha_2},$$

$$D_i = \frac{2h^3E_i}{3\left(1 - \nu_{ij}\nu_{ji}\right)}, \quad D_{ij} = \frac{2h^3G_{ij}}{3}, B_{ij} = 2hG_{ij}, \ B_i = \frac{2hE_i}{1 - \nu_{ij}\nu_{ji}}, \ \Lambda_i = 2hG_{i3},$$

$$i, j = 1, 2; \quad i \neq j$$

Consequently, we get two boundary value problems: defined by the equations (1), (3), and (2), (3).

For solving of the above mentioned boundary value problems by the indirect method of the boundary elements the Green's functions that can be found on the base of the Fourier method and the sequential representation of the Dirac delta-function (as the limit of the sequence of the delta-like functions [2, 3]) are considered. Hence the solvable boundary value problem for finding the proper Green's functions consists of the equations (3) in the domain $\Pi : 0 \le \alpha_1 \le l, 0 \le \alpha_2 \le 2\pi,\ \Omega \in \Pi$ where

$$q_s = \mathrm{T}_s^r \delta_{\varepsilon 1}\left(\alpha_1, \alpha_1^r\right) \delta_{\varepsilon 2}\left(\alpha_2, \alpha_2^r\right) e^{i\omega t}, \quad s = \overline{1,3},$$

$$m_p = \mathrm{T}_{3+p}^r \delta_{\varepsilon 1}\left(\alpha_1, \alpha_1^r\right) \delta_{\varepsilon 2}\left(\alpha_2, \alpha_2^r\right) e^{i\omega t}, \quad p = 1, 2, \tag{4}$$

$$\delta_\varepsilon\left(\xi, \xi^r\right) = \begin{cases} \dfrac{1}{2\varepsilon} g\left(\dfrac{\left|\xi - \xi^r\right|}{\varepsilon}\right), & \left|\xi - \xi^r\right| \le \varepsilon, \\ 0, & \left|\xi - \xi^r\right| > \varepsilon, \end{cases}$$

and the following homogeneous boundary conditions (equivalent to the conditions of simple support) on its boundary $\partial\Pi$:

$$w = 0, \quad M_n = 0, \quad N_n = 0, \quad u_\tau = 0, \quad \gamma_\tau = 0 \tag{5}$$

The principal moment of the solution is that the integral equations are formulated on the external boundaries of the shell as well as on the internal boundaries (contours of cutouts) according to the procedure demonstrated in the paper [1]. Integral equations are solved numerically using collocation method.

Conclusions

1) On the base of the found solutions making the proper combinations of the constructed integral equations and their cores the solution for the arbitrary mixed cases of the boundary conditions can be obtained. Arbitrary combinations of the magnitudes $w(\alpha),\ u_n(\alpha),\ \gamma_n(\alpha),\ u_\tau(\alpha),\ \gamma_\tau(\alpha),$ $Q_n(\alpha), M_n(\alpha), N_n(\alpha),\ M_\tau(\alpha), N_\tau(\alpha)$ on each contour can be considered. Moreover, within each contours (both internal and external) arbitrary mixed boundary conditions on each subsection are also allowed.
2) Within the problem statement it is not indispensably that the external boundary is fixed. The one among the internal contours can be considered somehow

fixed or number of them simultaneously by putting the proper input magnitudes equal to zero.

3) The key equations take into consideration shear displacements and include all inertial components. This allows to investigate in the better quality different types of vibrations caused by different character of boundary excitation in case of anisotropic materials.
4) In the framework of these solutions the cases of the contours with the corner points and the holes degenerates into the cracks can be analysed. Though on the stage of construction integral equations on the base of the potential theory the condition of the smoothness of the contours is required, but on the stage of getting the numerical solution by the collocation method the curves are discretised, and, consequently, this limitation looses its actuality.
5) On the computational stage it is important to investigate the convergence and optimal values of the approximation parameters within each particular case for getting the efficient enough numerical results, because if the geometrical configurations of the contours and anisotropy of the material are sharp, the solutions will be more sensitive, and will need more care and accurate treatment.
6) The demonstrated in the paper scheme indicates reasonable agreement with known results for the classical partial cases.

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2. *Burak Ya.J., Rudavsky Yu.K., Sukhorolsky M.A.*. Analytical mechanics of the locally loaded shells. – Lviv: Intelect-Zakhid, 2007. – 240 p.
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КОЛИВАННЯ ОРТОТРОПНОЇ ЦИЛІНДРИЧНОЇ ОБОЛОНКИ З МНОЖИНОЮ ОТВОРІВ ДОВІЛЬНОЇ КОНФІГУРАЦІЇ

В рамках уточненої моделі, яка враховує поперечні зсуви, побудовано розв'язок задачі про усталені коливання ортотропної циліндричної оболонки з довільною кількістю отворів довільної геометричної форми та розташування. Торці оболонки також вважаються довільної конфігурації. Розглянуто довільні гармонічні в часі граничні умови на контурах отворів, а також на зовнішній границі оболонки. Розв'язок побудовано на основі непрямого методу граничних елементів. Функцію Гріна знайдено на основі послідовнісного підходу до зображення дельта-функції Дірака та методу рядів Фур'є. Крайову задачу зведено до системи інтегральних рівнянь, яку розв'язано методом колокацій.