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ON SPECTRA OF ENERGY-DEPENDENT STURM-LIOUVILLE OPERATORS

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The aim of the talk is to discuss spectral properties of the quadratic operator pencil

$$T(\lambda) := \lambda^2 - \lambda B - A \qquad (1)$$

in the Hilbert space $L_2(0,1)$, where A denotes an operator acting by $Ay := -y'' + q(x)y$ subject to the Dirichlet boundary conditions and B is the operator of multiplication by *2p*. Here *p* is real-valued functions from $L_2(0,1)$ and *q* is real-valued distribution from $W_2^{-1}(0,1)$.

The *spectrum* of the operator pencil T is the set $\sigma(T)$ of all $\lambda \in \mathbb{C}$ such that $T(\lambda)$ is not boundedly invertible. A number $\lambda \in \mathbb{C}$ is called an *eigenvalue* of T if $T(\lambda)y = 0$ for some non-zero function $y \in \operatorname{dom} T$, which is then the corresponding *eigenfunction*.

Vectors $y_1, \dots y_{m-1}$ are said to be associated with an eigenvector y_0 corresponding to an eigenvalue λ if

$$\sum_{i=0}^{j} \frac{1}{i!} T^{(i)}(\lambda) y_{j-i} = 0, \quad (j = 1, \dots, m-1).$$

The maximal length of a chain composed of an eigenvector y_0 and vectors associated with it is called the *algebraic multiplicity* of an eigenvector y_0. The *geometric multiplicity* of eigenvalue λ is the dimension of the null-space of the operator $T(\lambda)$. The eigenvalue is said to be *algebraically (geometrically) simple* if its algebraic (geometric) multiplicity equals to one.

Let us consider the functions $y_1 := y$ and $y_2 := \lambda y$. By means of these functions we can recast the spectral problem of interest as that for the linear operator $\mathrm{L} := \begin{pmatrix} 0 & I \\ A & B \end{pmatrix}$ defined on the proper domain.

Proposition 1. The spectrum of the operator pencil T coincides with that of the operator L accounting multiplicities.

Using this proposition we obtain the spectral properties of T from those of L.

We consider the operator L in the space $\Pi := (\mathsf{D},[\cdot,\cdot])$, where $\mathsf{D} = H_{1/2} \times H$, H is the space W_2^1 and H_α is the domain of the operator $|A|^\alpha$ for any positive α. Here $[\cdot,\cdot]$ is an inner product defined as

$$[x, y] = (Ax_1, y_1) + (x_2, y_2)$$

The space Π is a Pontryagin space [1,2], i.e., the space with indefinite inner product. The operator L is self-adjoint in Π. Thus using the spectral theory for self-adjoint operators in Pontrygin spaces we derive the spectral properties of L and thus those of the operator pencil of interest *T.*

*Theorem 1***.** Let κ be the number of negative eigenvalues of the operator A. Then

- the spectrum of the operator pencil T is discrete;
- there are at most κ non-simple real eigenvalues, and their algebraic multiplicities do not exceed $2\kappa + 1$;
- the non-real spectrum of the pencil T is symmetric with respect to the real axis and consists of at most κ pairs of eigenvalues λ and $\overline{\lambda}$ of finite algebraic multiplicity, moreover, the root subspaces corresponding to λ and $\overline{\lambda}$ are isomorphic.

If the operator A is positive, then the spectrum of the operator pencil T is real and simple.

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ПРО СПЕКТРИ ЕНЕРГОЗАЛЕЖНИХ ОПЕРАТОРІВ ШТУРМА–ЛІУВІЛЛЯ

Мета доповіді дослідити спектральні властивості операторів Штурма-Ліувілля з потенціалами, залежними від спектрального параметру. Використовуючи допоміжні функції, спектральну задачу, що розглядається, можна звести до спектральної задачі для деякого лінійного оператора, що є самоспряженим у спеціально визначеному просторі Понтрягіна. Використовуючи спектральну теорію для самоспряжених операторів у просторах Понтрягіна, ми отримаємо властивості спектру лінійного оператора і, відповідно, вихідної спектральної задачі.