ABOUT FINITE AXIOMATIZABILITY OF CONGRUENCE-FAITHFULL ACTS

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Let S be semigroup with 0 and 1 and let M be unitary centred left S−act. Category of all such left acts we denote by $S-Act$.

An equivalence relation ρ on M is called left S−congruence on M if $(x_1,x_2)\in\rho$ implies $(sx_1,sx_2)\in\rho$, $\forall s\in S,\forall x_1,x_2\in M$. Let Δ denote identity congruence on S.

Definition 1. Left annihilator of $m\in M$ is S−act congruence $Ann(m)=\{(s,t)\in S\times S\mid sm=tm\}$.

Respectively left annihilator of S-act M is S−act congruence $Ann(M)=\{(s,t)\in S\times S\mid \forall m\in M, sm=tm\}$.

Definition 2. S-act M is called faithfull if $Ann(M)=\Delta$.

The first order language of left S−acts denote by L_S. Set of all sentence of language L_S that are true for class of acts Ψ is called theory of class and denoted by $Th(\Psi)$. Model of theory T is class of acts such that all sentence of this theory are true on it; denote by $Mod(T)$ class of all such models. Class is called axiomatizated if $Mod(Th(\Psi))=\Psi$. Axiomatizated class Ψ of acts is called finitely axiomatizated if $Th(\Psi)$ is finite. Next theorem is analog of result from [1] for acts.

Theorem 1. For monoid S next conditions are equivalent:

1) Class of congruence-faithfull unitary left S-acts is finitely axiomatizated;

2) There exists finite set $F\subseteq S\times S$ such that S-act M is faithfull if $\forall(a,b)\in F\ \ \exists x\in M:\ ax\neq bx$;

3) There exists $T\subseteq(S\times S)\setminus\Delta$ - finite set that has nonempty intersection with any congruences on S;

4) S contains such finite collection A of minimal congruences that any non-identity congruence on S contains at last one congruence from A.

1. Скорняков Л.А. Конечная аксиоматизируемость класса точных модулей.// Colloquium Mathematicum, (1979), P.365-366.

ПРО СКІНЧЕННУ АКСІОМАТИЗОВНІСТЬ КОНГРУЕНЦ-ТОЧНИХ ПОЛІГОНІВ

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