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CHARACTERIZATION OF THE MACRO-CANTOR SET IN COARSE CATEGORY

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The well-known Cantor set is defined as follows:

$$2^{\omega} = \{x = \sum_{i=1}^{\infty} k_i 3^{-i} \mid k_i \in \{0,2\}\}.$$

In the asymptotic geometry there is a natural its analog called the macro-Cantor set:

$$2^{<\mathrm{N}} = \{x = \sum_{i=1}^{\infty} k_i 3^{i} \mid k_i \in \{0,2\}\}.$$

The macro-Cantor set in the asymptotic geometry plays the same role as does the Cantor set in zero-dimensional topology. The main result is a characterization of the macro-Cantor set up to coarse equivalence.

We define a multi-map $\Phi : X \to Y$ between two metric spaces to be *large scale uniform* if for every $\delta \in [0,\infty)$ the number

$$\alpha_{\Phi}(\delta) = \sup\{diam(\Phi(A)) : \quad A \subset X diam(A) \le \delta\}$$

is finite.

A multi-map $\Phi : X \to Y$ between metric spaces is called a *coarse equivalence* if $\Phi(X) = Y$, $\Phi^{-1}(Y) = X$ and both the multi-maps Φ, Φ^{-1} are large scale uniform.

Metric space (X,d) is called *asymptotically zero-dimensional*, if for any $a > 0$ there exists a totally bounded a-disjoint cover of X .

Theorem. A metric asymptotically zero-dimensional space (X,ρ) is coarsely equivalent to the macro-Cantor set if and only if there exists a number $a > 0$ such that the following conditions hold:

1) $\forall n \in N \quad \exists d > 0 \quad \forall x \in X$ the set $U_d(x)$ cannot be covered by less then n balls of radius a .

2) $\forall d > 0 \quad \exists m \in N \quad \forall A \subset X$ if $diam(A) < d$ then A can be covered by m balls of radius a .

Using this result one can easy prove that any finite exponent and finite power of the macro-Cantor set is coarsely equivalent to it.

1. *J. Roe,* Lectures on coarse geometry. University lecture series. V. 31. - Providence, R.I. : American Mathematical Society. – 2003. - vii, 175 p.

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