Інститут прикладних проблем механіки і математики ім. Я.С. Підстригача НАН України
Рада молодих науковців і спеціалістів

Конференція молодих учених «ПІДСТРИГАЧІВСЬКІ ЧИТАННЯ – 2010»
м. Львів
25-26 травня 2010 року

Секція: СУЧАСНІ ПРОБЛЕМИ МАТЕМАТИКИ
http://www.iapmm.lviv.ua/chyt2010/materials/pc2010-02-M-20.pdf

УДК 512.55

ULTRACLOSEDNESS OF SOME CLASSES OF DIFFERENTIAL MODULES

Melnyk I.O.

Ivan Franko National University of Lviv, ivannamelnyk@yahoo.com

Many recent studies have focused on prime submodules and their analogues. However, few investigations have been focused on the differential analogues of prime submodules [1], [2]. The talk is devoted to the investigation of some properties and constructions of differential modules preserved by ultraproducts.

Let R be an associative differential ring with the set of pairwise commutative derivations $\Delta = \{\delta_1, \delta_2, \dots, \delta_n\}$, and let M be a differential R-module with the set $D = \{d_1, d_2, \dots, d_n\}$ of module derivations consistent with the corresponding ring derivations.

A differential submodule N of M is called *quasi-prime* if there exists an m-system S of the ring R and an Sm-system S^* of the module M such that N is maximal among differential submodules with $N \cap S^* = \varnothing$.

A differential submodule N of M is called *differentially prime* if M/N is differentially prime, i. e. if $\operatorname{Ann}_l(K) = \operatorname{Ann}_l(M/N)$ for every non-zero differential submodule K of M/N.

A differential module M is called *$d-MP$-module* if one of the following equivalent conditions holds:

1. Every quasi-prime submodule N of M is prime;
2. Every quasi-prime submodule N of M is radical;
3. For every prime submodule N of M the submodule
$$N_{\#} \overset{df}{=} \left\{ x \in M \,\middle|\, x^{(i_1,\dots,i_n)} \in N \text{ for all } i_1,\dots,i_n \in \mathbb{N} \cup \{0\} \right\}$$
is prime;
4. The radical of any differential submodule is a differential submodule.

Let M be a $d-MP$-module over a commutative differential ring R, let S be a multiplicatively closed subset of R.

Theorem 1. If every differentially prime differential submodule of M is prime, then M is a $d-MP$-module.

Let I be an infinite set, and let $\mathcal{U}$ be a nonprincipal ultrafilter over I. Suppose that for each $i \in I$ N_i is a submodule of the differential R_i–module M_i, then one can construct a submodule of $\prod_{i\in I} N_i/\mathcal{U}$ of the ultraproduct $\prod_{i\in I} M_i/\mathcal{U}$, which is an ultraproduct of a family of submodules $\{N_i\}_{i\in I}$ with respect to $\mathcal{U}$ [4].

Theorem 2. Let N_i be a quasi-prime submodule of the differential R_i – module M_i, $i \in I$. The ultraproduct $\prod_{i\in I} N_i/\mathcal{U}$ of the family $\{N_i\}_{i\in I}$ of quasi-prime submodules is a quasi-prime submodule of $\prod_{i\in I} M_i/\mathcal{U}$. Every quasi-prime submodule of the differential module $\prod_{i\in I} M_i/\mathcal{U}$ is an internal ultraproduct of some family of quasi-prime submodules $\{N_i\}_{i\in I}$ of $\prod_{i\in I} M_i/\mathcal{U}$.

Theorem 3. The ultraproduct of any family of $d-MP$ –modules with respect to the nonprincipal ultrafilter is a $d-MP$ –module.

1. *Khadjiev Dj., Callialp F.* On a differential analog of the prime-radical and properties of the lattice of the radical differential ideals in associative differential rings // Tr. J. of Math. – 1996. – **20**, №4. – P. 571–582.
2. *Verschoren A.* Differential rings with polynomial identities I (Quasiprime ideals in noncommutative differential rings) // Bull. Soc. Math. Belg., Ser. B. – 1983. – **35**, №2. – P. 227–245.
3. *Мельник І. О.* Sdm -системи, диференціально первинні та диференціально примарні модулі // Наук. вісник Ужгородського університету. Сер. матем. і інформ. – 2008. – Вип. 16. – С. 110-118.
4. *Eklof P.* Ultraproducts for algebraists: Handbook of Mathematical Logic. - Ed. J. Barwise. – North-Holland, 1977. – P. 109-140.

ПРО УЛЬТРАЗАМКНЕНІСТЬ ДЕЯКИХ КЛАСІВ ДИФЕРЕНЦІАЛЬНИХ МОДУЛІВ

Досліджуються деякі властивості диференціальних модулів, що зберігаються при переході до ультрадобутків. Встановлено, що клас $d-MP$ -модулів є ультразамкненим.